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THE UNIVERSITY OF ALBERTA

CONCEPTS OF NUMBER OF GRADE I STUDENTS

by

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A Thesis
Submitted to
the Faculty of Graduate Studies

In Partial Fulfillment
of the Requirements for the Degree
Master of Education

Department of Educational Psychology

Edmonton, Alberta
October, 1964

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled, "Concepts of Number of Grade I Students" submitted by Jonas Joseph Fornwald, B.A., B. Ed., in partial fulfillment of the requirements for the degree of Master of Education.

ACKNOWLEGEMENTS

The writer wishes to acknowledge a considerable debt to the many people whose co-operation and assistance made possible the completion of this thesis.

In particular, thanks are due to Dr. J. D. Ayers, who offered helpful criticism and encouragement at all times. Thanks are also due to the other members of the committee, Dr. L. D. Nelson and Dr. W. B. Dockrell, for their suggestions.

The writer also acknowledges his indebtedness to Joseph Quinn for his assistance in the testing program.

Finally, appreciation is expressed to the principal and staff and students of the following schools, Mayfield, Meadowlark, Grovenor and McArthur, without whose kind assistance and excellent co-operation this study would have been impossible.

ABSTRACT

The primary purpose of this study was to determine to what extent grade one children have developed some of the basic concepts of number. The study concerned itself with the following specific concepts:

1. Cardinal number
2. Ordinal number
3. The conservation of number
4. Arithmetical operations
5. Place value as applied to the decimal system of notation.

In addition, the study attempts to clarify the nature of the relationship between the child's knowledge of basic number concepts and sex, intelligence, socio-economic status, program of instruction and chronological age.

A test consisting of five subtests, each designed to measure a particular concept, was constructed and administered individually to a sample of eighty grade one students. These children ranged in age from six years five months to seven years five months; they had all attended school for a period of nine months prior to the time the tests were administered in June, 1964. The students were selected from four schools, two from the West Jasper Place Public School System and the other two from the Edmonton Public School System. At the present time, all four schools are in the city of Edmonton, Alberta, Canada.

A summary of the children's performance on each of the various subtests was completed. The responses of these children to each of the

test items were factor analyzed in an attempt to determine what factors were being measured and to what extent the various subtests were independent of each other. Further analysis of childrens' scores, based on only twenty test items, was made in an attempt to clarify the relationships noted earlier.

The most important findings of this study are thus summarized:

1. Chronological age contributed significantly to the child's concept of numbers. The number concepts of children aged seven years to seven years five months were superior to the children aged six years five months to six years eleven months.

2. Sex contributes significantly to the child's concept of number. The number concepts of girls were superior to the boys'.

3. The socio-economic status and intelligence of the student plus the type of instructional program are factors which do not significantly contribute to the child's knowledge of basic concepts of number.

This study has many implications for educational programs. Some of these implications are presented together with some suggestions for further research.

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CHAPTER I

INTRODUCTION

Mathematics has always held a principal position in the curriculum of the schools, chiefly because it has been a necessary component in the foundation of a liberally educated person. Through the ages mathematics has played an important role in precisely this sense, and today it has greater potentiality than ever for contributing to liberal education. But if the educated person, the desirable end product of our elaborate educational process, is to understand mathematics, he must develop an understanding of the basic concepts of number at the beginning of his elementary mathematics program.

Modern programs in elementary mathematics are attempting to establish within the child a foundation of basic concepts of number through a sequential and systematic developmental process. No assumptions are made that the beginning school child has any of the basic concepts established and thus these concepts, beginning with very primitive ideas of number are developed sequentially and systematically. Formal work with number symbols and drill activities are undertaken only after these basic conepts have been established.

The fundamental ideas of number include the concept of one-to-one correspondence, the concept of cardinal number, the concept of ordinal number, the concept of invariance of number, an understanding of the basic arithmetical operations and a thorough knowledge of place value as used in the decimal system of notation. All modern programs

of mathematics attempt to establish this foundation during the child's first year of formal education.

The major purpose of this study is to determine to what extent the basic concepts of number have developed in grade one students; in addition, the variables, sex, intelligence, socio-economic status, the effects of different instructional programs and chronological age will be controlled to determine their relationship to the child's continuous development of the basic concepts of number.

DEFINITION OF TERMS

Concepts. A concept may be defined as an idea, or set of related properties that can be predicated of two or more individual items. A concept requires both abstraction and generalization, the first to isolate the relevant defining property, the second to recognize that it may be descriptive of several objects.

Concepts of Number. Number concepts are generalizations about the relationships that exist between certain kinds of data. They are the basic ideas of number, divorced from objects, actions, or number symbols, and exist as abstraction in the child's mind.

One-to-one Correspondence. Objects when matched one-to-one are labelled as being in one-to-one correspondence. These objects may be alike or totally dissimilar. When one-to-one matching is performed the total number of objects in one group is equal to the total number of objects in the second group.

Cardinal number. The cardinal property of number is an abstract

property assigned to a set of objects whose nature remains constant regardless of the shape, size or arrangement of objects comprising the group. For example, the cardinal number, "six" refers to any collection of six objects.

Ordinal Number. The ordinal property of number refers to the rank number of a unit in a series. It is the positional value such as third, fourth, or fifth object in a series.

Conservation. This term refers to the concept that a given total remains invariant regardless of the arrangements or rearrangements of its parts. For example, in comparing quantities of liquid, the child who believes that the amount of liquid has increased when it is poured into a different shaped container has not acquired conservation. His concept of total quantity is influenced by his perceptions, the liquid now appears to take up more space, therefore he believes the total quantity to be greater.

Arithmetical Operations. This term refers to the basic operations of addition, subtraction, division and multiplication which are taught in the elementary arithmetic program.

Place Value. This term will be used to refer to the fact that the value assigned to a digit depends upon the position of the digit in the numerals used to represent a number.

Grade one student. The term will designate a student who is in his initial year of formal education.

Ginn Series. The term will refer to an elementary arithmetic program; it is an "Arithmetic We Need" Series published by Ginn &

Company. The name of the grade one test is "Teaching Numbers We Need".

Ten major objectives of this program, for grade one students, are:

1. Rote counting
2. Rational counting
3. Cardinal number
4. Ordinal Number
5. The structural meaning of numbers in the decimal system with a base of ten.
6. Money - a familiarity with the common coins and their usage.
7. Measurement
8. Mastery of addition and subtraction facts and using proper operational symbols.
9. The ability to combine three groups and to read and write number stories involving addends.
10. Working with equal groups as readiness for the ultimate systematic study of multiplication and division.

Gage Series. The term will refer to another elementary arithmetic program; it is a "Seeing Through Arithmetic" Series published by W. J. Gage Limited. The name of the grade one text is "Numbers We See".

The five major objectives of this program, for grade one students, are:

1. Correspondence: relating one object to one object, one-to-two, two-to-one, one-to ten, recognizing groups of two, three and four objects.

2. Number relationships: finding how groups of objects (up to ten) can be built up from small groups and how these larger groups can be broken up into various arrangements of the smaller groups.
3. Number system: grouping by tens as well as by tens in combination with ones, leading up to an appreciation of the written notation of our number system.
4. Measurement: developing the concept of a unit and standard unit.
5. Money: recognize the smaller coins and know relationships between them and what they will buy.

OVERVIEW

Chapter II will be devoted to a review of the literature relative to the development of the basic concepts of number. Chapter III will present the theoretical framework, postulates and hypotheses. Chapter IV will outline the sample, the procedure followed in conducting the study and the statistical procedures to be used in analyzing the results. Chapter V will present the findings of the investigation and the final chapter will contain the conclusions, together with some recommendations and suggestions for further research.

CHAPTER II

A REVIEW OF RELATED LITERATURE

Basic to any attempt to improve the teaching of arithmetic is a knowledge of the way in which young children form number concepts and how the teaching of arithmetic and number experiences in the primary elementary school affect this growth. Although arithmetic itself has been the subject of research for many years, much of the reported material lacks of theoretical basis. It seems, therefore, that an attempt should be made at the outset to sketch a psychological theory of interpretation on which a systematic body of knowledge, more directly useful to teachers, may be built.

Buswell, (1951), in a review of the previous twenty years of research in arithmetic, had similar thoughts in mind when he stated that too much time has been spent in dealing with incidental problems, for example, analyses of textbooks and curricula, comparisons of specific techniques of teaching given topics, and far too little on the thinking being done by children as they learn.

During the 1920'3 and 30's, the "Committee of Seven", under the chairmanship of Washburne (1939), was set up to investigate the grade placement of topics in the arithmetic curriculum in the United States. With teaching methods and time allocation controlled, each topic was introduced at three or four consecutive grades in a population of over 30,000 children in sixteen states. A battery of intelligence tests and foundation tests, (to estimate mastery of previous knowledge required

for a given topic) was given. The topics were then treated in class. Finally, a battery of attainment tests was given to measure mastery and retention of the new topics. A minimum age at which eighty per cent of the children had mastered each item was determined. It was found that when tested at Grade VI, the best performance came from those who began formal arithmetic in Grade I (age 6+); followed by those who began at Grade II (age 7+); and finally by those who commenced at Grade III (age 8+). However, in addition, if a child did not begin formal arithmetic until Grade II or Grade III and instead was given informal number activities and number games, his later performance was very much superior to the child who began formal arithmetic in Grade I. The Committee recommended that addition facts with sums of ten and under, and easy subtraction facts, should not be attempted before the mental age of seven to eight years. Up to this level the curriculum should consist of "informal experience and activities to give children real concepts of number and space relations, without any systematic drills." (p. 309)

Churchill's (1958) research demonstrated that with appropriate teaching methods and number experiences, the requisite level of conceptual understanding can be developed as early as five to six years. The implication is rather that until this understanding is developed, at whatever age, it is a waste of time teaching such operations as $7 + 2 = 9$. Premature arithmetic teaching, she suggests, may set up barriers to the proper understanding of numbers when the child is ready. Williams (1958) recommends a readiness program, consisting of the basic

processes of ordination and cardination, which should assist the child in acquiring operational number concepts. When these are attained, and not before, the child is ready to proceed to more formal work.

In the past, two methods of teaching arithmetic have been advocated--the mechanical method and the meaningful method. Up to the 1930's articles on the teaching of arithmetic stressed an approach derived from the connectionistic, stimulus-response laws of behaviourist psychology. This theory proposed that the more frequently the pupil practised drill, the more firmly "fixed" the number facts became in his mind.

The Gestalt school of psychology then became popular and very different implications for a teaching method resulted. The emphasis was upon "structured wholes" and the inter-relationships of parts of the whole. Applied to arithmetic, it implies that the understanding of number relations and of the total structure of arithmetic should be aimed at: an approach obviously more suited to problem solving than to drill; drill was deliberately neglected as being mechanistic and therefore inimical to the development of number concepts.

Although the controversy between the advocates of mechanical and meaningful teaching has continued for about twenty-five years, Miller (1957) in his review of research points out that the relative superiority of the meaning method was as yet undetermined and that "little research has been initiated to justify the great emphasis on the approach." (p. 45)

Prior to Piaget's work, there was little known about the logical

properties of numbers in relation to the corresponding psychological development of number concepts. Few theoretical approaches offered much insight into the sequential and developmental aspects of number understanding and hence their applicability to teaching method, was limited.

PIAGET'S RESEARCH ON THE DEVELOPMENT OF THE CONCEPT OF NUMBER

In the past ten years, there has been a marked awakening of interest throughout the world in the work of the Swiss psychologist Jean Piaget. Piaget's (1952) experiments were designed to investigate the beginnings of understanding of number in the minds of children of ordinary ability between the ages of four and eight. He wanted to determine the developing number of relevant capabilities considered more subtle and basic than those involved in the familiar elementary operations of counting, of rote addition, of subtraction, etc. The results of his experiments have influenced, in recent years, the type of arithmetic curriculum developed for the primary grade child.

Piaget conducted his experiments with children of pre-school and primary school age in a type of "clinical method," which was a combination of observation and interview. His experiments were designed to test the child's knowledge and to identify the stages of development in such concepts as conservation of quantity, cardinal and ordinal correspondence, the properties of cardinal and ordinal number, seriation, the additive composition of numbers and their relationship to each other, the co-ordination of relations of equivalence and the multiplicative composition of numbers, and the additive and multiplicative composition

of relations and the equalizations of differences.

Piaget observed that children of about six years of age display a spontaneous interest in numbers since they have usually learned to count. He claims that these children have only a vague notion of what the concept of number is. At this stage judgments about problems involving numbers are to a large extent determined by what the child perceives, so that if a perceived structure is changed, the numerical judgments made about the situation will be likely to change. This point was demonstrated when a row of six blocks side by side were observed by children to be smaller than the same number of blocks when they were spread out into an extended line. At the same time, the young child's judgments about quantities appear to be determined by perceived characteristics, rather than a logical notion which obeys laws of conservation. This stage, which is bound to a particular perceptual point of view, is referred to by Piaget as the stage of "global comparisons." It is followed by an "intuitive" stage in which the child starts to realize that judgments of quantity and number cannot be made simply in terms of perceived attributes of objects, or sets of objects, which remain invariant under perceptual transformations. The child begins to free itself from the purely perceptual field, although his cognitive activity is still bound to judgments about objects in the perceptual field.

The third stage in which judgments become completely "operational" is referred to by Piaget as the stage of "concrete operations". Judgments are no longer bound to perceived patterns or particular perceptual points of view. Operations and judgments start to manifest stability,

self-consistency, and reversibility, but they can only be performed on perceived objects, not in the abstract.

According to Piaget, operations which are necessary conditions for an understanding of numbers are, the ability to deal with the equivalence of cardinal classes in terms of one-to-one correspondence, and the ability to deal with order relations such as "greater than" and "less than". This implies that in order to understand what a number is, a child must be able to manipulate and make judgments about perceived objects in such a way that (1) the order, or pattern arrangements, of elements in a group of objects does not influence judgments about the number of objects present, and (2) the child presumably can arrange objects in a series according to an obvious criterion such as size and should be able to deal with ordinal correspondence between different series. Then the child has reached the stage of "concrete operations".

Piaget's theory of number can be summarized as:

1. The first stage, he referred to as "global comparisons" is restricted to immediate perceptual relationships. The average child age four to five is representative of this stage. The basic concepts of conservation, seriation, ordinal and cardinal number are absent.

2. The intermediate stage, classified as "intuitive" is characterized by partial notions of conservation. At this stage, children of age five to seven can perform simple numerical operations but they are superficial, meaningless and valueless, for the foundation is not there.

3. The third stage is the stage of "concrete operational

thought". Number has become an operating concept in the child's own thought fully abstract from perceptions and its vicissitudes. Generally, Piaget's children reached this stage between six and a half to eight years of age.

The work of Piaget stands out because it suggests a clear picture of the psychological processes involved at various levels of intellectual behavior, and at the same time offers an account of the way in which these are elaborated in the course of maturation and of uncontrolled experience. However, his work has been criticized for (1) failing to specify the number of subjects used in any one investigation, (2) his statistical methods are questionable, and (3) he frequently bases generalizations on the behavior of but one or two children and fails to say whether a particular type of observed behavior is universal, typical or merely found occasionally, at any particular stage or age of development.

RESEARCH BASED ON PIAGET'S THEORY

Piaget's theory of the development of number concepts has served as a basis for research projects in all parts of the world. A study in Edmonton by Brace (1963) attempted to determine to what extent pre-school children have developed some of the basic concepts of number. A test consisting of six subtests, each designed to measure a particular concept, was administered individually to a sample of one hundred and twenty-four pre-school children ranging in age from five years four months to six years five months. Based on his findings, the following

conclusions can be made: (1) the pre-school child's ability to count is not a reliable criterion of the extent to which he has developed a knowledge of the true concept of number; (2) a complete understanding of cardinal number is required before the child can have real facility with ordinal number; (3) a child requires a thorough knowledge of the concepts of cardinal number, ordinal number and rational counting before he can understand the structure of our numeration system; (4) the influence of older siblings and sex fail to add to the child's understanding of basic ideas of number; (5) the child's concepts of number is significantly influenced by the environment and increases with chronological age; (6) some children had reached Piaget's second stage in the development of number concepts. There was little evidence, however, for the third stage in the children tested in this investigation.

Holmes (1963) specifically investigated three areas with tests adapted from Piaget's experiments. These areas are: (1) the developmental levels of cardinal and ordinal number concepts; (2) the change in the developmental level of number concepts as the child increases in age; (3) the association between cardinal number concepts and ordinal number concepts of young children. The test was administered individually to one hundred and forty-four pre-first grade children from middle-class and upper middle-class families in San Jose, California. Her findings indicate that: (1) pre-nursery and the nursery children show little understanding of number; (2) the development of several aspects of number is characterized by unevenness at the kindergarten and the pre-school levels; (3) in certain number concepts, kindergarten children

are relatively mature, while in other number concepts they are poorly developed. Her results generally agree with Brace's study; however, due to her select sample, generalizations must be limited.

Dutton (1963) using a sample of two hundred and thirty-six kindergarten children selected from eleven schools in the Los Angeles area attempted: (1) to discover specific number concepts possessed by pre-school children at the time of kindergarten entrance; (2) to secure some indication of pupil achievement in arithmetic after one year in kindergarten. He administered the Metropolitan Readiness, forms S and R, for his pre and post tests. He concluded that one-third of the children entering kindergarten were ready for systematic number work. In addition, there was a significant gain in number concepts through the informal teaching of arithmetic during the child's year in a kindergarten.

A major study consisting of a large sample with a wide age range was performed by a Canadian psychologist; Dodwell (1960) attempted to investigate and evaluate the theory of cognitive development in terms of the three stages suggested by Piaget. He used two hundred and fifty Kingston, Ontario students who were attending kindergarten, Grade I or Grade II, ranging in age from five years, one month to ten years, one month. His instrument consisted of fifty-four questions which were divided into five subtests. His results confirm the three stages of cognitive development. Also, the types of responses described by Piaget for all three stages were found, and his "intuitive" stage was quite prevalent. The most striking responses in this category, given

by several children, was to count the poker chips when one of the two rows was extended, but still to judge that there were different numbers of chips in the two rows. The conclusion is obvious, counting "per se" is no guarantee that a child grasps what the concept "cardinal number" is, or how it applies to a concrete situation.

Wohlwill (1960) investigated the sequence through which an individual passes in the course of the development of a concept of number. He divided seventy-two students equally into six age levels, from four years to seven years by half-term intervals; they were all attending various kindergarten and primary schools in Geneva, Switzerland. His findings agree with Piaget's distinct stages, but he conceptualized the developmental process in a slightly different though clearly related terms, proceeding from an initial, preconceptual stage, in which number is responded to purely in perceptual terms without any symbolic mediation, through an intermediary stage, in which the mediational structures are developed and eventually a final stage, where the superordinate structures representing the number concepts in the abstract are refined.

A most significant study with a group of multi-racial children was conducted by Hyde (1959). He compared their performance on Piaget's number tests. The sample consisted of forty-eight European children, forty-eight Arab, twenty-four Indian, and twenty-four Somalis. They were divided into three equal groups, age six, age seven, and age eight. His most significant finding was that non-European children tend to acquire a basic numerical understanding considerably later than Europeans. However, the European children were privileged while the other children

were under-privileged and culturally deprived. He concluded that cultural factors influence the development of operational reasoning. In addition, the answers given by Arabic children were often verbatim translations of the answers given in French by Piaget's children at comparable levels. The results also confirm Piaget's development stages.

An experiment to test the applicability of Piaget's theoretical formulations to educational practice was developed by Churchill (1958). Two groups, each consisting of eight children aged five, were matched for their understanding of number concepts from the results of a test situation similar to Piaget's. The experimental group participated in a special program of number experiences for a period of four weeks. After retesting, the results indicated a highly significant improvement as compared with a control group, when judged by the number of questions which were answered at an "operational" level. The sequence of stages as identified by Piaget were also confirmed in this study.

Coxford (1964) measured the effects of instruction on the stage placement of children in Piaget's seriation experiments. Twenty-four children ranging in age from three years, six months to seven years, five months were matched by chronological age with a control group. The experimental group received instruction in groups of two for ten to fifteen minutes during four sessions. The chronological ages of the total sample corresponded on the average to Piaget's predicted stages. The results showed that the experimental children tested at Stage II made significant gains in stage placement by means of instruction over the control children who received no instruction but the same results were not evident for

those on Stage I. There appears to be two reasons for the seemingly conflicting results. They are: (1) the mean IQ for the Stage I group was 120; the mean IQ for the Stage II group was 135, (2) a certain chronological age must be reached before a child can advance a stage on the basis of instruction alone. Further research is needed to clarify these results.

Coward's (1951) test which included eleven subtests attempted to evaluate the child's number ability ranging from rote counting to processes of addition and subtraction. It was administered individually to twenty-five girls and twenty-five boys at each of the four age levels from three through six years. Some of the more pertinent results are:

1. The ability to count greatly increases at the six year old level.
2. Rote counting is superior to object counting.
3. Concepts of measure are limited until age five.
4. Girls show superiority in all concepts.
5. The ability to solve problems increases with age.
6. Number ability of children in the upper three occupational groups is consistently superior to that of subjects in the lower three occupational groups.

Coward suggested that the girls superiority in all concepts is partially explainable by the established linguistic superiority of girls. He also infers that school experience caused the smaller differences between occupational groups which appeared at age six.

Martin (1954), using a test adapted from Coward, attempted to

determine the competences of a child at a given age in number concepts. His sample consisted of fifteen boys and fifteen girls at each age level from three through seven years, a total of one hundred and fifty children. The results demonstrated a steady increase in number ability with age, sex differance in favor of the girls, differences in favor of those children in the upper occupational groupings and a significant relationship with intelligence.

A study which attempted to determine specifically the nature of the process at work in the development of the concept of conservation of number was conducted by Wohlwill and Lowe (1962). Their investigation was a form of nonverbal matching, from a sample type learning experiment, preceded and followed by verbal questions to measure the child's understanding of the conservation principle. The seventy-two kindergarten children were divided into four conditions of training which consisted of: (1) reinforced practice on conservation, (2) dissociation of biasing perceptual cues, (3) inferential mechanisms based on the recognition of the effects of addition and subtraction of the elements, (4) a control group. The results indicated an over-all increase in nonverbal conservation responses from a pre to a post test but they show no significant differences attributable to the conditions of training. There was, however, a consistent tendency for the addition and subtraction condition to yield the most improvement in conservation. This suggests further research is required to evaluate the benefits of giving the children considerable experience in simple additions and subtractions during the period when conservation generally becomes functional.

Feigenbaum (1963), using a sample of ninety students ranging in age from four to seven years, attempted to test whether Piaget's explanation emphasizing operations occurring in an invariable age-stage system is sufficient to account for success or failure in understanding the principle of conservation. He found that "stages of development" in the acquiring of the conservation concept were not defined by definite age barriers, but rather descriptive general trends. The data also indicated that children's grasp of the conservation concept varies with their IQ rating and with the nature of the concrete experimental operations, for example, changing the number of beads.

The research studies reviewed tend to agree with Piaget's model of three developmental stages in the attainment of basic concepts of number. The studies also indicate that environment, racial factors, sex and education influence the developmental stages of number concepts.

CHAPTER III

THEORETICAL FRAMEWORK

Cognition, similar to digestion, is an organized process. Every act of intelligence presumes some kind of intellectual structure, some sort of organization within which it proceeds. At birth, the only organizations available are the congenital sensorimotor structures such as reflexes or instincts. As the organism interacts or adaptively copes with the environment, it continually reorganizes its intellectual structures.

Piaget conceives the adaptive interaction between organism and environment to involve two complementary processes, assimilation and accommodation. Intellectual assimilation is not different in principle from the more primary biological assimilation. In both cases the essential process is that of fitting an experience (event) to fit the individual's present cognitive structures.

Accommodation, the counter-part of assimilation, represents the concomittant change in cognitive structure in order to adjust to recognized variations in environmental stimuli. In the course of this dual adaptive process of assimilation and accommodation the ready made reflexive structures of the new born infant become progressively transformed through differentiations and co-ordinations into broad, effective, information processing structure characteristics of adult intellectual processes.

The functional characteristics of the assimilatory and

accommodatory processes are such that the possibility of cognitive change is insured, but the magnitude of any given change is always limited. The organism adapts repeatedly and each adaption provides the basis for its successor. Structures are not infinitely modifiable, however, and not everything which is potentially assimilable can in fact be assimilated by a certain organism at a precise point in its development. The reverse is true, the subject can incorporate only those components of reality which its present structure can assimilate without drastic change.

The following is an outline of the stages of intellectual growth as defined by Piaget (1951). The three general stages of intellectual growth, with certain sub-stages, and approximate age levels in each case are:

Stage I - Non-operational: Birth - 2 years.

Stage II - Pre-operational: (a) Transduction - 2 - 4 years
(b) Intuitive - 4 - 7 years

Stage III - Operational: (a) Concrete - 7 - 11 years
(b) Formal - 11 years and over

Now, a brief explanation of each stage.

Stage I: (Non-operational; 0 - 2 years)

He refers to this stage as the sensorimotor period. It consists chiefly of simple hereditary reflexes, redefined, and supplemented by habits learned through trial and error.

Stage II (a): (Pre-operational - Transductive; 2 - 4 years)

Here, the child is introduced to signs and symbols, particularly words and images. These expose him to a world which extends far

beyond his momentary range of vision, a stable and consistent world compared to the changing world of perception he had previously known. Memory thus becomes possible, as the abstraction from action to thought is made. At the same time, with the development of language objects are symbolized by a thought process which can be retained in the mind.

Stage II (b): (Intuitive; 4 - 7 years)

To demonstrate the characteristics of this stage,the following example is used. Piaget gave his children some brown beads and a much smaller quantity of white ones; both groups of beads were wooden and belonged to the concept "wooden beads". The children maintained that there were more brown ones than white ones and while agreeing that both brown and white were wooden, would then go on to make the claim that "a necklace made out of brown beads would be longer than one made out of wooden beads, because there are more brown ones than wooden ones." Not until the age of seven were these children able to hold two concepts in mind simultaneously and thus grasp the logical and mathematical truth that the part is never greater than the whole.

Stage III (a): (Concrete; 7 - 11 years)

During this period of concrete operations, various operational groupings become consolidated, the internalized actions or operations become reversible as the principle of conservation is attained. However, the child is still capable of only a limited form of reasoning, which is tied to his own concrete experience.

Stage III (b): (Formal; 11 years and over)

This is called the abstract or formal operational stage. At early adolescence, the ability to perform abstract operations becomes apparent. Unlike the "child", who concerns himself only with actions in progress, the adolescent thinks beyond the present and forms theories about everything. This reflective thought exists from the age of 11 - 12 years, when the individual becomes capable of reasoning on the basis of simple assumptions which have no reality, to conclusions which follow logically from them. He becomes interested in the necessary validity of an inference, as opposed to agreement of the conclusions with experience. He can consider hypotheses which may or may not be true and work out what would follow if they were true.

Upon re-examining the period extending approximately from birth to eight years of age, we find that the child's intellectual structures develop from very elementary activities to the stage where the concepts of number are functional at the concrete operational level. Initially, during this period, the home environment provides the necessary experiences for the progressive redefining and expansion of the child's structures. In addition, we can assume that the home environment will be significantly influenced by the socio-economic and educational level of the parents. Therefore, it can be concluded that the socio-economic status of the family will affect the children's intellectual development.

Jones (1954), in his review on socio-economic classes and

intelligence, provided the following conclusions:

1. The evidence was significant for social class differences in intelligence.
2. Environmental opportunities substantially influence intellectual development.
3. The correlation between measures of parental social status and children's intelligence was about .50.

Brace (1963) concluded that the development of the child's concept of number may be significantly affected by the environment in particular the socio-economic level of the home. Churchill (1958) suggested that the type of environment can accelerate the progress in developing a concept of number in children.

After age six, the child is stimulated by the school environment. Here, formal activities associated with the basic concepts are introduced. In contrast to the home environment, the children of all socio-economic levels are subjected to common activities at school. Therefore, the children should demonstrate a trend towards homogeneity of achievement in the basic concepts of number. This may be a valid conclusion for a classroom or even a number of schools. However, at the present time, all schools do not use the same arithmetic programs or the same methods of instruction. The two most commonly used programs are: (1) "Arithmetic We Need", a Ginn Series; (2) "Seeing Through Arithmetic", a Gage Series. The two programs have similar objectives but they differ in their approach in developing these objectives.

Initially, the Ginn Series stresses the immediate attachment of

symbols to objects. Secondly, the basic concepts of number are developed with symbols, rather than with concrete objects, by means of drill activities which stress memorization.

The Gage Series delays symbolic arithmetic operations until the basic concepts of number are developed by concrete methods. An understanding of the processes involved are stressed rather than a memorization of the activities involved. The concrete manipulation of objects is a basic requirement for this program. The rationale for teaching the basic concepts of number in this manner is that it helps the child to pass progressively from concrete thinking to the more abstract modes of thinking. This agrees with Inhelder (Bruner, 1960) who suggested that "the first two years of schooling should be given over to activities which will set the pattern of later intellectual functioning." (p. 47)

In considering the whole child, in addition to environmental factors, there are two hereditary factors, natural intellectual endowment and sex. These appear to influence the development of structures necessary for the understanding of the basic concepts of number. Piaget (1950) found that not every child followed the specific stages, as he theorized, in their intellectual development. Piaget's bright child achieved a particular level earlier than the average child, and the dull child failed to reach the final stages. Grant's (1938) analysis showed a positive relationship between number ability and ratings of intelligence rating and various arithmetical skills. A significant relationship between number ability and ratings of intelligence was established by Martin (1954).

In various studies, the results appear inconsistent as to the influence of sex differences on specific aspects of intellectual development. In the study by Coward (1951), which was later replicated by Martin (1954), girls were superior in all number concepts and at all age levels from three to seven years except at age four. No significant differences were found to exist between the number concepts of playschool (pre-school) boys and girls, in the study by Brace (1963) or by Mascho (1960) with grade one students. With older children, Terman and Tyler (1954) found that on a general battery of achievement tests, girls excel in English, spelling, writing and art. Boys tend to do better in arithmetic, especially arithmetic reasoning, history, geography and science. All the differences were small.

In summary, children continually improve their intellectual functioning by interacting with the environment through the processes of assimilation and accommodation. In addition, hereditary and environmental factors determine the stages of development at a specific time and the rate of progression from initial to the final stages of intellectual development. The formation of intellectual structures necessary for the comprehension of the concepts of number are only specific aspects of the total intellectual development.

POSTULATES

The adaptive interaction between the child and his environment fosters a continuous intellectual development.

1. As the child increases in chronological and mental age, its

concepts of number progressively change from a "transductive" to an "intuitive" to a "concrete" and then to a final "formal" operational stage.

2. Hereditary, environmental factors and planned learning influence the development of the intellectual structures required for the formation of concepts of number.

HYPOTHESES

1. There is no significant difference in the number knowledge of children in the age group six years five months to six years eleven months and those in the age group seven years to seven years six months.

2. There is no significant difference between the number knowledge of males and females.

3. There is no significant difference in the number knowledge of children in the Gage Series instructional program and those in the Ginn Series instructional program.

4. There is no significant difference in the number knowledge of children with high IQ ratings and those with low IQ ratings.

5. There is no significant difference in the number knowledge of children of high socio-economic status and those of low socio-economic status.

CHAPTER IV

EXPERIMENTAL DESIGN

The population consisted of all the grade one students who were attending the West Jasper Place Schools and the Edmonton Public Schools during the 1963-64 school year. Two different school systems were selected in order to add another variable, program of instruction, to the experimental design. The West Jasper Place schools use the "Seeing Through Arithmetic" series published by Gage while some Edmonton schools generally use the "Arithmetic We Need" series published by Ginn for their grade one arithmetic instructional program. The preliminary sample selected consisted of four schools, Mayfield and Meadowlark in West Jasper Place and Grovenor and McArthur in Edmonton. A total of 323 grade one students attended these four schools; 148 in the West Jasper Place schools and 175 in the two Edmonton schools. Twelve classrooms were represented, six from each system. The four schools were arbitrarily selected for the following reasons: (1) their instructional programs met the requirements of the experimental design; (2) the four schools compared very favorably in their representation of students from all socio-economic levels.

SAMPLE

The final sample consisted of an equal number of boys and girls whose ages varied from six years five months to seven years five months. A distribution of the sample in reference to age and sex is presented in

TABLE I

AGE AND SEX OF CHILDREN INCLUDED IN SAMPLE

Age* (Years and Months)	Male	Female	Total
6 - 5	1	3	4
6 - 6	4	1	5
6 - 7	5	2	7
6 - 8	2	5	7
6 - 9	2	5	7
6 - 10	2	1	3
6 - 11	4	3	7
7 - 0	4	5	9
7 - 1	5	3	8
7 - 2	4	5	9
7 - 3	2	3	5
7 - 4	4	3	7
7 - 5	1	1	2
TOTAL	40	40	80

*Age was computed to the nearest month as of June 1, 1964.

Initially, one-half of the sample (40 students) was selected from the West Jasper Place schools using the Gage Series in the following manner. The 148 students were stratified into three dimensions, sex, age by months, and socio-economic status into two groups. A number was then assigned to each student. Finally, with the aid of a table of random numbers a total sample of 40 students was selected from all the various subdivisions.

The other half of the sample was selected by matching the randomly selected group with 40 students from the two Edmonton schools using the Ginn Series. The students were matched on the four variables, sex, age,

socio-economic status and I.Q. Where it was feasible, person by person matching was used. The remainder of the sample was selected on the basis that the variable means for all cells were not significantly different.

The total sample was divided into eight cells with ten students per cell on the basis of age, sex and instructional programs. Table II presents a complete illustration of the sample.

TABLE II

SAMPLE SUMMARY
(N=80; 10 per cell)#

		PROGRAM OF INSTRUCTION								
Age Groups		Gage Series				Ginn Series				
		Male		Female		Male		Female		
		$\bar{X}$	S	$\bar{X}$	S	$\bar{X}$	S	$\bar{X}$	S	
1 6 yrs. 5 mos. to 6 yr. 11 mos.	Age*	80.0	2.0	80.1	2.0	80.2	2.1	80.2	1.9	40
	I.Q.	109.6	8.7	109.0	6.5	108.9	10.0	109.5	11.9	
11 7 yrs. to 7 yrs. 5 mos.	Age	86.0	1.5	85.9	1.6	86.0	1.8	86.0	1.6	40
	I.Q.	108.9	9.4	108.6	10.0	109.9	10.6	109.0	12.4	
TOTALS		20		20		20		20		80

*Age in Months
#Each cell contains 5 high and 5 low socio-economic status students

The sample was stratified as illustrated in Table II in order to perform a three way analysis of variance. Within each cell, the means and standard deviations are shown for age and I.Q. It is readily apparent that means and deviations for the four cells at each age group level are not significantly different; however, since the two age groups were set up to be different, a "t" test between the two means with the smallest difference was performed to determine if these groups were significantly different. In addition, in order to test differences in I.Q., means for all eight cells, a "t" test was applied to the two means with the largest difference. The Ginn Series students (Edmonton sample) are generally more variable on I.Q. as indicated by the larger standard deviations. However, when the homogenity of variance for the eight cells was tested by a conventional "F" test, no significant differences were found. Table III presents a summary of the two "t" tests.

The "t" tests indicate a significant difference between the two age means which have about a six month (5.7) age difference. This agrees with the experiment design. The mean I.Q.'s do not differ for the eight cells. It can be concluded therefore from Tables II and III that within the structure of the three way analysis of variance the cells are not significantly different on the variables of age, I.Q. and socio-economic status.

TABLE III

SUMMARY OF "t" TESTS BETWEEN THE TWO AGE GROUPS AND BETWEEN THE TWO MEAN I.Q. SCORES

	N	$\bar{X}$	S	$\bar{X}$ diff.	"t"	Sign
Age						
Group 11 (7 yrs. to 7 yrs. 5 mos.)	10	85.9	1.6	5.7	6.79	$P < .01$
Group 1 (6 yrs. 5 mos. to 6 yrs. 11 mos.)	10	80.2	2.1			
I.Q.						
(Largest cell X)	10	109.9	10.6	1.3	.28	N.S.
(Smallest cell $\bar{X}$)	10	108.6	10			

Intelligence Ratings

The two groups of students had I.Q. ratings based on different tests and norms. The Edmonton students had a Detroit Beginners I.Q. rating which was computed from local norms with a mean of 100 and standard deviation of 16. The West Jasper Place students had Lorge Thorndike, Level 1 ratings; their I.Q. ratings were based on the publishers norms, a mean of 100 and a standard deviation of 16.

Due to a possible difference between norms, it was decided to compute the mean and standard deviation for the present grade one West

Jasper Place students. Based on 793 students, a mean of 105.5 and a standard deviation of 12.94 was calculated. Thus, in order to equalize the I.Q. ratings for the two school systems, the regression equation $100 + \left(\frac{X_1 - 105.5}{12.94}\right) \times 16$ was applied to all Lorge Thorndike ratings. In addition, all students with I.Q. ratings below 85 or above 132 were eliminated from the preliminary sample.

Socio-Economic Status

All students were classified into two groups, high socio-economic status and low socio-economic status. The father's occupation was the basis for classification. All children whose fathers were in professional, managerial, sales, or clerical positions were classified as high socio-economic status, while those whose fathers were semi-skilled, unskilled or unemployed were classified as low socio-economic status. In general all white collared workers were regarded as being of high socio-economic status, while all others were placed in the low socio-economic category.

INSTRUMENT

In reviewing various tests used in previous experiments, Brace's instrument (1963) and Piaget's experiments were most helpful in preparing the instrument used in this study to measure the basic concepts of manner. In addition, suggestions as to the type of problems and suitable language were gained from Grade one teachers.

A rough draft of the test was administered to nine grade one students who did not form part of the final sample. On the basis of

the pilot run, the test was revised. Some of the questions which appeared unsuitable were omitted while others were restated in more appropriate language.

The final draft of the test consisted of five subtests, each designed to evaluate the child's knowledge of a specific concept. The number of items per subtest ranged from five to ten. A copy of the test is provided in Appendix A. The following is a description of the five subtests together with the names of the concepts they were designed to measure.

I. The Cardinal Property of Number

This subtest was designed to measure the child's ability to recognize a group of objects consisting of a number named by the experimenter. The subtest consisted of two parts. In the first part, the child was presented with eight cards of red buttons ranging in number from three to ten and arranged on the cards in domino patterns. The child was asked to point to the card with three buttons on it, ten buttons, etc. For the second part, small cubical blocks were used to structure the ten groups, ranging from three to twelve blocks. The groups were randomly aranged and the blocks within each group had random patterns. Again the child was asked to point to a group of five blocks, eight blocks, etc. All students performed identical tasks.

II. The Ordinal Property of Number

The child's knowledge of ordinal number was assessed with the following objects: a toy fireman and fireman's ladder, an arranged

parking lot on which fifty toy cars were parked in five rows of ten cars each, and twelve toy horses arranged in single file and supposedly running a race. Each child was asked to place the fireman on the fourth step on the ladder, locate the eighth car in a third row, find the horse that is in fifth place, etc. This subtest consisted of ten items.

III. Conservation of Number

This subtest attempted to evaluate the child's grasp of the concept of invariance of number. For example, the child was presented with two equal groups of red and blue blocks (12 of each). The two groups were arranged in a one-to-one correspondence with each other. When the experimenter concluded that the child had perceived the equality of groups, he spaced the blue blocks one block apart. Then, the child was asked if there were still as many red blocks as blue blocks. Conservation of continuous (liquids) and discontinuous (beads and marbles) quantity was also assessed. For example, two equal sized glasses, with equal amounts of liquid were placed before the child. When the child was convinced that the liquids were equal, the experimenter poured the blue liquid into a tall, slender glass. The child was then asked if there was more blue liquid than red liquid. An attempt is thus made to measure the child's conservation of continuous quantity during perceptual changes. Similar experiments were conducted with marbles and beads.

IV. Arithmetical Operations

The first eight items of this subtest were designed to measure the child's ability to perform simple addition and multiplication operations with concrete objects. For example, for the operation of addition, the child was asked to confirm the equality or non-equality of two groups of chips without counting; the child's group consisted of fifteen chips in two groups $(9 + 6 = 15)$ while the experimenter's group consisted of fifteen chips in three subgroups $(5 + 5 + 5 = 15)$. An example of the operation of multiplication consisted of asking the child to point to one group out of a total of five groups, which had just enough chips to enable the experimenter to give each of the five soldiers two chips $(2 \times 5 = 10)$. The last two items, a subtraction question and an addition question were performed mentally in the absence of concrete materials. In addition, the last three items required that the students explain in the form of a number story his manner of attaining a solution. For example, six take away four is equal to two $(6 - 4 = 2)$.

V. Place Value in the Decimal System of Notation

This subtest was designed to determine if the child has the concept of how the numbers are built up in the decimal system of notation. For example, the child was asked to make groups of fourteen, forty-five, etc., from bundles of ten sticks and single sticks.

All subtests will be discussed further in the summary of the analysis.

PROCEDURE

The testing program was conducted in the four schools during the period from the 12th to the 23rd of June, 1964. The instrument was adminstered to individual students by the writer and an assistant. Testing was conducted at each of the four schools in a secluded room, free of interruptions.

Five testing stations, corresponding to the five subtests, were arranged on tables. With specifically marked bases for the placement of all test materials, the arrangements and special relationships for all items were consistent for every student.

Initially, when the student entered the room, he was allowed to spend two or three minutes looking at all test materials. Once the testing had started, it continued in sequence from the first subtest throughout all five subtests. The actual testing required from fifteen to twenty-five minutes per student.

The Analysis to be Used

All test items (1 to 41) were scored by assigning a one or a zero to each one, depending upon the child's ability to arrive at the correct answer or not. Based on the assumption that if the child could arrive at the correct answer, he had some knowledge of the concept involved, and was then assigned a score of one for that particular item. It was assumed that his knowledge of a particular concept was inadequate or inaccurate if he was unable to attain the correct answer, a zero score was

then given. The data were then punched on IBM cards and the following analysis was completed by means of the IBM 1620 computer.

The intercorrelations between all subtests scores, age and I.Q. scores were computed. Next, since the subtests were set up arbitrarily by the writer, it was decided to find the intercorrelations (phi-coefficients) between all items and then to factor analyze the phi-coefficients in an attempt to determine whether the variations revealed in the forty-one items could be accounted for in terms of a small number of categories, considerably less than the number of items. The principal axes method which maximizes the variances accounted for by each factor, was employed. The factors having been isolated by means of an eigenvalue-eigenvector solution, the axes were then rotated to permit useful psychological interpretation of these factors. If the subtests are indeed independent then it would be expected that the factors isolated by the analysis would correspond with great similarity to the subtests.

A three-way Analysis of Variance will then be applied to the test scores of the 80 students to determine the main effects and interaction for the three variables, sex, age and program of instruction.

CHAPTER V

SUMMARY AND ANALYSES OF RESULTS

The analyses of this study is presented in three sections within this chapter. They are:

1. Summary of Results for Total Sample

11. Factor Analysis

111. Testing of Hypotheses

I. SUMMARY OF RESULTS FOR TOTAL SAMPLE

A general summary of the performance of the total sample (80 students) on all items of the test will be presented in Table IV. This table represents the item difficulty as expressed by the percentage of children scoring correct answers on all forty-one items. No attempt will be made at this stage to differentiate between the various sub-groups represented by the sample, such as age, I.Q., program of instruction, sex or socio-economic status.

TABLE IV

DIFFICULTY OF ITEMS IN SUBTESTS
(N = 80)

Item Number	Per cent Successful	Item Number	Per cent Successful
1	100	21	81
2	74	22	54
3	88	23	75
4	66	24	70
5	91	25	45
6	61	26	54
7	40	27	70
8	40	28	70
9	93	29	59
10	60	30	63
11	75	31	36
12	76	32	54
13	52	33	43
14	31	34	84
15	93	35	63
16	68	36	76
17	81	37	75
18	53	38	65
19	79	39	79
20	58	40	71

The test for basic concepts of number was divided into five subtests. A summary of the performance of the total sample on each of the various subtests will be presented.

Results of Subtest I - Cardinal Number

This subtest measures the child's ability to recognize the cardinal number of objects in a group without counting. The groups ranged in size from three to twelve, both in pattern arrangements (buttons on a card) and in random arrangements (groups of blocks randomly arranged). This subtest consisted of eight items. Table V presents a summary of the results.

TABLE V

SUMMARY OF RESULTS OF SUBTEST I
CARDINAL NUMBER
(N = 80)

Score	Per cent Successful
1	1.3
2	5.0
3	10.0
4	8.7
5	17.5
6	22.5
7	18.7
8	16.3

The scores although ranging from one to eight tend to be skewed towards the maximum scores. Only 25 percent of the total sample attained

scores greater than four.

Results of Subtest II - Ordinal Number

This subtest, composed of ten items, was designed to evaluate the child's concept of ordinal number. The items were divided into three parts.

The first and third part attempt to evaluate the child's knowledge of ordinal number in one dimension, such as placing the fireman on the fourth step of the ladder, or locate the fifth horse in a row. The second part consisted of two operations; for example, the child had to locate the eighth car in the third row in a parking lot of fifty toy cars arranged in five rows of ten cars each. A summary of the results appear in Table VI.

TABLE VI

SUMMARY OF RESULTS ON SUBTEST II
ORDINAL NUMBER
(N = 80)

Score	Per cent Successful
1	0
2	1.3
3	3.7
4	7.5
5	10.0
6	22.5
7	20.0
8	12.5
9	12.5
10	10.0

The scores ranged from two to ten with slightly less than one-quarter (22.5%) of the students attaining a score of five. A major proportion of the students (42.5%) attained scores of six or seven. The remaining students (33%) gained maximum scores of eight, nine or ten.

Results of Subtest III - Conservation

This subtest was designed to measure the child's understanding of conservation. For example, the child was asked to match in a one-to-one correspondence a small toy rider with a large toy horse. When the child was certain that each horse had a rider, all horses were moved together into a large pile, and the riders were placed into a small pile. The child was then asked if there were more horses than riders. Other items required that the child place a small bead in his glass at the same time as the experimenter placed a large marble in his glass. The two glasses were exactly the same size and a total of twelve marbles and twelve beads were used with each child. The child was then asked if the experimenter had more marbles than he had beads. Slightly less than one-half of the students (46%) were affected by the perceptual differences and thus appear to allow perceptual differences to affect their concepts of conservation. Liquids were also used for two items in this subtest (see test, Appendix A). The results of the eight items from this subtest are summarized in Table VII.

TABLE VII

SUMMARY OF RESULTS OF SUBTEST III
CONSERVATION OF NUMBER
(N = 80)

Score	Per cent Successful
0	6.3
1	3.7
2	8.8
3	12.5
4	6.3
5	12.5
6	8.8
7	13.7
8	27.4

The students scores ranged from a zero to eight. A small proportion (6.3%) appeared to totally lack the concept of conservation while slightly more than one-quarter (27.4%) consistently demonstrated adequate knowledge of the concept. These students were not affected by perceptual changes. The remaining students' scores were spread unevenly between the two extremes.

The results of the items involving liquids indicated that spatial arrangement was interfering with the child's concept of conservation. When the contents from one of two equal containers of liquid was

poured into another container that was taller and small in diameter, the child believed that the quantity was greater because the level of the liquid was higher. Other students were not confused by the change in spatial arrangement because they generally explain their answer by stating that amount of liquid remained constant while the spatial arrangement changed. On both items, about fifty per cent of the students successfully applied the concept of conservation.

Results of Subtest IV - Arithmetical Operations

Six items in this subtest were designed to measure the child's ability to perform the operation of addition. In these items, the child was presented with two large groups of concrete objects which were subdivided into smaller groups, The child was then required to determine the equality or non-equality of the two large groups. Two other items indirectly attempted to evaluate the child's knowledge of the operation of multiplication. For example, the child was required to select one group of sticks from four different sized groups which would allow him to give three sticks to each of the two soldiers, $(3 \times 2 = 6)$. Two other items were designed to measure the child's ability to perform the operations of addition and subtraction without the aid of concrete objects. Then, items 34, 35 and 36 required the students to verbally state, in the form of a number story, their method of attaining a solution; for example, six plus three equals nine. Table VIII summarizes the results.

TABLE VIII

SUMMARY OF RESULTS OF SUBTEST IV
ARITHMETICAL OPERATIONS
(N = 80)

Score	Per cent Successful
1	1.3
2	5.0
3	7.5
4	7.5
5	17.5
6	17.5
7	11.3
8	15.0
9	13.7
10	3.7

The results are distributed from scores of one to ten with three-quarters of the students attaining scores of 5, 6, 7, 8 or 9. Only a small proportion, 3.7 per cent of the students, attained the maximum score on this subtest.

Results of Subtest V - Place Value

All the items are designed to evaluate the child's concept of the construction of our number system of notation. The child was required to arrange into groups of 14, 19, 45, 97 and 113, coloured sticks using

bundles of ten sticks and single sticks. Table IX summarizes the results.

TABLE IX

SUMMARY OF RESULTS OF SUBTEST V
PLACE VALUE
(N = 80)

Score	Per Cent Successful
0	13.7
1	6.3
2	8.8
3	18.7
4	43.7
5	8.8

The major proportion of the students (62.4%) successfully applied the concept of place value on three or four of a possible five items. However, this concept proved to be the most difficult, since 13.7 per cent demonstrated a complete lack of knowledge of the concept as measured by this test. One item in this subtest, number 41, required the students to apply the concept of place value beyond one hundred in the number system. Both instructional programs, Ginn Series and Gage Series, deal with the construction of the number system only up to one hundred; therefore, the students were required to apply the concept beyond their grade one instructional program. Only 8.8 per cent of the students

successfully applied the concept beyond one hundred.

II. FACTOR ANALYSIS

The intercorrelations were computed for the continuous variables. These variables include subtest scores, total test scores, age, and I.Q. for the total sample. The intercorrelations are presented in table X.

TABLE X

INTERCORRELATION MATRIX FOR CONTINUOUS VARIABLES
(N = 80)

Subtests	1	2	3	4	5	Total Score	Age	I.Q.
I	1.000	428	241	542	288	702	252	139
2		1.000	054	274	375	578	131	070
3			1.000	454	402	680	104	261
4				1.000	450	800	369	266
5					1.000	692	129	180
Total Score						1.000	285	237
Age							1.000	-023
I.Q.								1.000

Decimal points are omitted.

A correlation of .28 is required to be significantly different from zero at the .01 level of significance with 78 degrees of freedom on a two tailed test.

Some of the more important aspects of Table X are:

(1) The generally high correlations between the individual subtest scores and total test score;

(2) The low to moderate intercorrelations of subtests;

(3) The low correlation of subtest scores and total test scores with both age and I.Q.

All the intercorrelations of the responses to the forty-one test items plus six reference variables were then calculated. All intercorrelations were phi-coefficients since all variables were treated as dichotomous and were scored one or zero. The six reference variables were dichotomized in the following manner:

(1) Total Score - All students' scores were divided at the median; scores above 27.5 were scored as one and below 27.5 as zero.

(2) Sex - The students were separated into groups of male and female.

(3) Age - All students were classified into two groups, seven years and above and below seven years.

(4) Program - The students were divided on the basis of their selection from the two instructional programs, Gage Series and Ginn Series.

(5) Socio-economic Status - Based on the occupation of the student's father, they were divided into two classes, high and low socio-economic status.

(6) I.Q.- The students were divided into two groups, those above a median score of 108.5 and those below 108.5.

On the six variables each dichotomous group represents one-half (40 students) of the total sample.

The intercorrelations for the total sample are presented in Appendix B. The absence of large intercorrelations (phi-coefficients) between the items indicates that each question is to a considerable degree independent of other items; in addition, one must bear in mind that phi-coefficients tend to be smaller than Pearson Product Moment correlations. However, since individual items are correlated, an attempt should be made to isolate some basic dimensions which account for the item variances.

An appropriate statistical technique is factor analysis, the purpose of which is to define a multi-dimensional set of items in terms of a more parsimonious set of factors.

The Principal Components Method of Analysis

The correlations (phi-coefficients) were then subjected to the principal axes method of factor analysis to determine a relatively small number of factors which will account for the major portion of the variance. In the principal axes method of analysis, the first principal factor accounts for the maximum possible variance, each successive one accounting for a maximum of the then existing residual variance. In this manner, the initial correlation matrix is reduced to the smallest number of orthogonal factors.

The unrotated factor loadings for each item on each factor having an eigenvalue greater than 1.32 are shown in Appendix B. Table XI presents the unrotated factor loadings for the first five factors.

Varimax Rotation

To permit the most satisfactory psychological interpretation of the factors, the unrotated factor matrix became input for the Princeton Package Rotations. The three output rotated matrices, quartimax, equamax, and varimax, were than analyzed to determine which rotation appeared most meaningful. Although all three rotations were quite similar, the varimax method was selected for interpretation.

The varimax rotation not only does a better job of approximating the simple structure principles but it also tends to level to factorially invariant solutions. Kaiser's (1958) varimax method places emphasis on simplifying the columns of factors, of the factor matrix by maximizing the variance of the squared factor loadings. When the variance is at a maximum, the factor has the greatest interpretability in that its components tend towards unity and zero.

The rotated factor matrix for eleven factors is presented in Appendix B. The first five rotated factors are presented in Table XII.

Interpretation of Factors

In the rotated factor matrix, all items with loadings in excess of an arbitrary value of .5 were considered to be the items which define a particular factor.

TABLE XI

UNROTATED FACTOR MATRIX FROM THE ANALYSIS OF FORTY-ONE TEST ITEMS AND REFERENCE VARIABLES

	COMMON FACTOR LOADINGS					
Variables	I	II	III	IV	V	h^2
1	-385	082	-114	-082	-433	362
2	-399	472	083	-407	-028	555
3	-446	020	-467	-057	-019	421
4	-453	349	-250	-268	095	470
5	-363	-132	-201	-472	-214	458
6	-400	359	-208	-168	-022	361
7	-404	267	218	-280	257	427
8	-440	323	354	-202	238	521
9	-346	007	-423	-243	-276	434
10	-164	382	-264	034	-113	256
11	-293	296	-122	089	-283	276
12	-127	095	042	-041	-644	443
13	-188	383	314	-333	-172	421
14	-140	457	337	-042	-274	419
15	-228	-094	145	125	-580	434
16	-438	515	-062	180	124	509
17	-355	291	-004	443	-121	422
18	-374	235	-129	204	203	295
19	-370	-185	301	101	-013	272
20	-571	-501	078	009	-084	590
21	-347	-266	598	120	-235	618
22	-528	-478	-087	028	058	519
23	-505	-160	273	-115	-083	375
24	-580	-555	013	-126	-006	661
25	-588	-398	205	-012	052	549
26	-674	-358	280	-069	-011	666
27	-450	-075	-101	-485	007	454
28	-419	-007	-189	-198	-247	312

TABLE XI (Cont'd.)

UNROTATED FACTOR MATRIX FROM THE ANALYSIS OF FORTY-ONE TEST ITEMS AND REFERENCE VARIABLES

	COMMON FACTOR LOADINGS					
Variables	I	II	III	IV	V	h^2
29	-368	084	065	-004	139	166
30	-278	-243	-202	-333	-068	293
31	-587	145	249	-135	303	538
32	-276	025	018	-045	-046	081
33	-361	110	070	-184	468	400
34	-352	001	-406	-008	-017	289
35	-459	-125	-347	100	159	382
36	-442	-043	-161	303	-101	325
37	-646	125	-165	322	039	565
38	-492	249	-043	445	-090	512
39	-605	095	145	489	035	636
40	-666	062	010	459	-001	658
41	-300	-103	-201	089	270	222
Tot. Score	-832	-064	009	-012	093	705
Sex	-304	-269	-181	067	202	243
Age	-310	083	-082	-025	155	134
Program	-060	480	456	-069	051	449
S.E.S.	-080	075	067	006	-291	101
I.Q.	-151	-103	164	-181	024	094
Sum of Squares	8.571	3.446	2.572	2.488	2.216	19.293
Per cent of Common Variance	44.41	17.9	13.3	12.9	11.5	100

Decimal points have been omitted for factor loadings.

TABLE XII

ROTATED FACTOR MATRIX FROM THE ANALYSIS OF FORTY-ONE TEST ITEMS AND REFERENCE VARIABLES

	COMMON FACTOR LOADINGS					
Variables	I	II	III	IV	V	h^2
1	-029	174	055	408	050	203
2	097	054	326	313	127	233
3	253	098	-027	270	309	243
4	175	099	096	374	369	326
5	217	-081	188	410	231	311
6	094	034	101	212	448	266
7	125	048	607	080	040	394
8	092	062	762	-014	122	608
9	015	220	030	630	-101	457
10	-147	168	084	100	150	089
11	-002	225	167	191	041	117
12	060	012	-047	077	010	012
13	048	-080	119	034	024	025
14	-096	313	056	000	-146	132
15	210	008	-209	075	269	166
16	-031	222	269	046	727	653
17	-024	335	100	-151	571	472
18	064	103	179	013	717	560
19	316	206	116	139	018	175
20	738	169	008	139	-048	595
21	420	103	081	-105	-067	209
22	782	125	002	048	024	630
23	449	040	256	105	065	284
24	680	060	117	310	035	577
25	778	149	133	002	-000	645
26	757	139	224	077	017	649
27	301	-063	222	649	065	569
28	142	-002	163	389	170	227
29	010	132	604	070	102	398

TABLE XII (Cont'd.)

ROTATED FACTOR MATRIX FROM THE ANALYSIS OF FORTY-ONE TEST ITEMS AND REFERENCE VARIABLES

	COMMON FACTOR LOADINGS					
Variables	I	II	III	IV	V	h^2
30	250	-104	006	600	144	454
31	296	143	630	056	207	551
32	191	-043	-011	001	177	069
33	153	073	531	125	137	345
34	-064	384	-018	521	-051	426
35	222	431	258	193	-090	347
36	240	396	042	-114	043	231
37	144	792	116	326	119	782
38	155	739	-035	023	140	592
39	235	678	211	-115	193	610
40	301	750	123	014	144	689
41	235	251	092	054	075	135
Total Score	601	338	249	223	289	670
Sex	139	234	015	304	056	170
Age	-031	200	360	138	065	194
Program	-188	106	113	-064	103	074
S.E.S.	-023	045	093	-044	-064	017
I.Q.	083	008	472	-078	-477	463
Sum of Squares	4.648	3.645	3.202	2.917	2.632	17.044
Per cent of Common Variance	27.3	21.4	18.8	17.1	15.4	100.0

The first five factors in the rotated factor matrix were selected for interpretation. The remaining factors had a maximum of two or less items with significant loadings. Thus their interpretation would be difficult and perhaps not too meaningful. They will be disregarded from all further discussion.

The initial five factors isolated, as determined by items with factor loadings in excess of .5, will be interpreted in the following manner:

FACTOR I - Conservation

This factor which is the largest contributor to the total variance was defined by Items 20, 22, 24, 25 and 26 (see Appendix A). This factor which corresponds to subtest III attempts to measure the child's understanding of the invariance of number. For example, Item 25 - after the child is convinced that there is the same amount of blue liquid as red liquid in the two equal glasses, the blue liquid is poured into a tall, thin glass. The child is then asked if there is more blue liquid than red liquid.

Items 19, 21 and 23 which were also included in Subtest III had loadings above .30 on this factor, however, they did not reach the arbitrary interpretation value of .5.

FACTOR II - Place Value

This factor was defined by Items 37, 38, 39 and 40 which were designed to evaluate the child's understanding of the construction of the

decimal system of notation. For example in Item 39 - the child is asked to make a group of 45 sticks by using bundles of 10 sticks and single sticks.

Subtest V consisted of Items 37 to 41, inclusive; Item 41 is disregarded because only 9 per cent of the students successfully completed this item.

FACTOR III - Group Recognition - A (cardinal number)

This factor was defined by Items 7, 8, 29, 31 and 33 which attempt to measure the child's ability to recognize groups of 8, 9, 10, 12 and 15 in random arrangements. For example, in Item 31, the child is asked to determine without counting if the total of his three groups of five chips each (5+5+5 = 15) is greater than the experimenter's total of two groups of nine and six (9+6 = 15). See Appendix A for the remaining items.

FACTOR IV - Group Recognition - B(cardinal number)

This factor was defined by Items 27, 30 and 34 which attempt to measure the child's ability to recognize groups of 7 and 9 in random arrangements. For example, in Item 30, again the child is asked to determine without counting if his total number of jacks in groups of eight and one (8+1 = 9) is less than the experimenter's total of three groups, with each having three chips (3+3+3 = 9). The remaining items are illustrated in Appendix A.

FACTOR V - Ordinal number

This factor was defined by Items 16, 17 and 18 which attempt to

evaluate the child's understanding of ordinal number. For example, in Item 16 - the child is asked to locate the ninth horse in a simulated racing situation. The other two items are the same except the child must locate the fifth and eleventh horse.

The items which define the first five factors, together with their loadings, are presented in Table XIV. The exact nature of each item is presented in Appendix A.

The first two factors isolated by factor analysis corresponds to Subtests III (conservation) and V (place value). The remaining factors were not as consistent with original subtest groupings. Generally, all items from Subtest I, (Cardinal number), and Subtest IV (arithmetical operations), had their highest loadings on Factors III and IV, group recognition. For Subtest I, these results are expected, but Subtest IV, which was designed to measure the child's ability to perform arithmetical operations became an exercise designed to measure the child's ability to recognize groups of various sizes. These results are appropriate according to mathematical principles since only Items 35 and 36 in Subtest IV approach true arithmetical operations. The remaining items in Subtest IV should have been labeled group recognition as the factor analysis has indicated.

Due to the similarity of items which load above .5 on Factors III and IV, they are interpreted as recognition of groups. The varimax rotation separates these two orthogonal factors which appear to define a similar ability, group recognition. It is conceivable however that in an

oblique rotation all the items which presently load on Factors III and IV would form one second order factor due to their stability.

In Subtest II (ordinal number), Items 9 to 18 inclusive had loadings above .5 on five different factors, Factors V to IX. Items 9 and 15 are dismissed because over 90 per cent of the students successfully completed these items while Items 10, 11 and 12 load on three different factors. Items 13 and 14 and the variable, program, define Factor VI by their significantly high loadings. Upon studying the students' results, it became evident that the students in the Gage Series were more successful on these two items than the students in the Ginn Series; however, their limited success was not due to their lack of knowledge of ordinal number but they appeared to lack flexibility in applying the concept when the directions were changed. For example, when the students were asked to change their starting position from the top left corner (Items 11 and 12) to the bottom right corner (Items 13 and 14), their response patterns varied from incorrectly starting to count the rows from the top right rather than the bottom right or initially, they determined the correct row and then proceeded to count incorrectly the cars from left to right rather than right to left.

Factor VI cannot be interpreted as defining the concept of ordinal number because the items which have high loadings on this factor are closely related to the Gage Series. The children in this program had more drill in applying the concept of ordinal number in different situations. The concept of ordinal number will be designated by Items 16, 17 and 18 as Items having loadings above .5 on Factor V.

For purpose of interpreting the basic concepts of number only twenty test items are represented in the first five factors; the remaining twenty-one items were discarded for the following reasons:

(1) Items 1, 5, 9 and 15 failed to satisfactorily discriminate between students; over 90 per cent of the students successfully completed these items;

(2) The loadings on the remaining items failed to meet the arbitrary level of .5.

In testing the hypotheses an estimate of the students' knowledge of the concepts of number was based on twenty of the original forty-one test items. These twenty items loaded above .5 on the first five factors and these factors are each interpreted as defining specific number concepts. The remaining six factors defined by only one or two items having loadings above .5 appeared to be uninterpretable. Therefore, the students' scores which were based on twenty rather than forty-one items were more valid estimates of their knowledge of the basic number concepts.

The specified items in Table XIII will be used to compute each student's total score. A student who successfully completed all the above items would have attained a perfect score of 20, for each item correctly answered he was assigned a score of one and a zero for an incorrect response. The scores for the total sample will then be used to test the hypotheses. Appendix C presents each student's total scores, plus his age, sex, I.Q., socio-economic status, instructional program and individual factor scores.

TABLE XIII

ITEMS DEFINING FACTORS

Item Number	Factors				
	I	II	III	IV	V
1					
2					
3					
4					
5					
6					
7			61		
8			76		
9					
10					
11					
12					
13					
14					
15					
16					73
17					57
18					72
19					
20	74				
21					
22	78				
23					
24	68				
25	78				
26	76				
27				65	
28					
29			60		
30				60	
31			63		
32					
33			53		
34				52	
35					
36					
37		79			
38		74			
39		68			
40		75			
41					
Total	5	4	5	3	3 = 20

III. TESTING THE HYPOTHESES

The following three hypotheses to be tested are:

1. There is no significant difference in the number knowledge of children in the age group six years five months to six years eleven months and those in the age group seven years to seven years six months.

2. There is no significant difference between the number knowledge of males and females.

3. There is no significant difference in the number knowledge of children in the Gage Series instructional program and those in Ginn Series instructional program.

A model I, 3-way analysis of variance as illustrated in Winer (1962), was used to test these hypotheses. The students' scores (see Appendix C) were based on twenty test items; these scores were analyzed to determine the main effects and interactions for the three variables, age, sex and instructional program. An illustration of the stratification for these three variables is presented on page 32 of this study.

The results of the analysis of variance is presented in Table XIV.

TABLE XIV

THREE-WAY ANALYSIS OF VARIANCE

	S.S.	df	M.S.	Obtained F Value	Significance
Age					
Students 7 years and above vs. Students below 7 years	110.45	1	110.45	5.16	$P < .05$
Sex					
Male vs. Female	88.2	1	88.2	4.12	$P < .05$
Program					
Gage Series vs. Ginn Series	1.80	1	1.80	.084	N.S.
Interactions					
Age and Program	22.05	1	22.05	1.03	N.S.
Age and Sex	8.45		8.45	.39	N.S.
Program and Sex	33.8		33.8	1.58	N.S.
Age, Program and Sex	8.45		8.45	.39	N.S.
Experimental Error (within cells)	1348.05	63	21.40		

The results of Table XIV indicate two significant differences; they are:

(1) The older students, seven years to seven years five months had superior number knowledge in comparison with students from the age of

six years five months to six years eleven months;

(2) The number knowledge of female students was superior to male students. The effects of different programs did not significantly affect the students number knowledge as measured by twenty test items. The three variables failed to significantly interact in any of the four arrangements.

A "t" test will be used to evaluate the following hypotheses:

(1) There is no significant difference in the number knowledge of children with high I.Q. ratings and those with low I.Q. ratings.

(2) There is no significant difference in the number knowledge of children of high socio-economic status and those of low scio-economic status.

Again, an estimate of the students' number knowledge is based on the scores (see Appendix C) they attained on twenty test items.

The effects of intelligence and socio-economic status on the students number knowledge is presented in Table XV .

TABLE XV

SIGNIFICANCE OF DIFFERENCE BETWEEN: (1) HIGH AND LOW I.Q.; (2) HIGH AND LOW SOCIO-ECONOMIC STATUS

	N	$\bar{X}$	S	Mean Difference	t	Significance
I.Q.						
High	40	12.42	4.6	.85	.86	N.S.*
Low	40	11.57	4.2			
Socio-economic status						
High	40	10.5	6.2	1.45	1.2	N.S.*
Low	40	11.95	4.2			

*a two-tailed "t" test of significance

Table XV indicates that there is no significant differences between the two I.Q. groups or the two socio-economic status groups. A student's I.Q. or socio-economic status rating did not significantly affect his knowledge of number concepts.

SUMMARY OF FINDINGS

The principal findings in relation to the hypotheses are briefly summarized.

Hypotheses 1 - There is no significant difference in the number knowledge of children in the age groups six years five months to six years eleven months and those in the age groups seven years to seven years six months.

This hypothesis was rejected. The differences, favouring the older students were found to be significant.

Hypothesis 2 - There is no significant difference between the number knowledge of males and females.

This hypothesis was rejected. The differences, favouring the girls were found to be significant.

Hypothesis 3 - There is no significant difference in the number knowledge of children in the Gage Series instructional program and those in the Ginn Series instructional program.

The hypothesis was accepted, the differences were not found to be significant.

Hypothesis 4 - There is no significant difference in the number knowledge of children with high I.Q. ratings and those with low I.Q. ratings.

This hypothesis was accepted, the differences were not found to be significant.

Hypothesis 5 - There is no significant difference in the number knowledge of children of high socio-economic status and those of low socio-economic status.

This hypothesis was also accepted, the differences were not significant.

CHAPTER VI

SUMMARY AND IMPLICATIONS

The major purpose of this study was to determine to what extent grade one children have developed some of the basic concepts of number. The study concerned itself with the following specific concepts:

1. Cardinal number
2. Ordinal number
3. The conservation of number
4. Arithmetical operations
5. Place value as applied to the decimal system of notation.

In addition, the study attempts to clarify the nature of the relationship between the child's knowledge of basic number concepts and sex, intelligence, socio-economic status, program of instruction and chronological age.

The instrument which was designed to measure grade one students' knowledge of the above specific concepts consisted of forty-one items. The test was administered to a sample of eighty students in the age range from six years five months to seven years five months. These children who were selected from four schools appeared to represent typical grade one students in urban school systems.

CONCLUSIONS

The final measure of the students' knowledge of number concept was attained on the basis of their performance on twenty items. The significance of these scores were tested by an analysis of variance and "t" tests

according to the experimental design. The following conclusions are based on the findings:

1. The child's ability to develop a knowledge of the basic concepts of number is directly related to his chronological age. In addition, it is the best predictor of the child's ability to attain these number concepts.

The older groups of students (seven years to seven years five months), demonstrated a superior knowledge of number concepts when compared to a similar group of grade one students between the age of six years five months. This finding is significant since the two groups were matched on sex, I.Q., socio-economic status, program of instruction, and length of time spent in grade one. Moreover, the difference between age means for the two groups was a meager six months.

This conclusion supports Piaget's theory that the child's development of basic number concepts is highly correlated to his chronological age.

2. The girls between the ages of six years five months and seven years five months have a superior knowledge of number concept in comparison to boys within a similar age range. The significance of this conclusion is opposed to that of Brace (1963). He found no significant difference between the number concepts of pre-school boys and girls.

There are at least two hypotheses for the superiority of girls after spending nine months in grade one. They are:

(a) We can accept that this finding is due to the established

linguistic superiority of girls; or

(b) We can hypothesize that female teachers reinforce the girls more than the boys and thus the girls attained a greater knowledge of the concepts of number. All teachers in this study were female. The study by McNeil (1964) on teaching reading appears to support this hypothesis. He found that boys were superior to girls in beginning reading under the neutral conditions of programmed instruction, however, these same boys did not maintain their superiority when placed under the instruction of female teachers.

3. Grade one students are not affected by the different instructional programs. There was no significant difference in number concepts of children in the Gage or Ginn instructional programs.

This conclusion is consistent with the objectives of each program; however, it should be mentioned that the teachers were free to add material, or choose not to follow the method of instruction as suggested by the course outlines. The variable, type of program must be rigidly controlled in a future study.

4. The variable of intelligence does not significantly affect the child's ability to develop a knowledge of the basic concepts of number. The number concepts of the high I.Q. students were not significantly different from the number concepts of the low I.Q. students. This conclusion may be due to a combination of the following reasons: (1) the unreliability of intelligence tests with

students at this age level; (2) the differences in I.Q. means for the two groups may not have been great enough to permit accurate specification of the role of intelligence in the attainment of number concepts.

There was no significant difference between the two I.Q. groups on the basis of the twenty items which measured five concepts. However, it was conceivable that a difference between the two I.Q. groups could exist on individual concepts. Students' scores were computed for each concept from the items which loaded above .5 on the first five factors, see Appendix C. Individual "t" tests were applied to all five concepts to determine the effect of intelligence. The groups on one concept, Group Recognition - A were significantly different at the .05 level on a two-tailed test. The high I.Q. students were superior. The possibility of this finding was suggested from the results of factor analysis. The variable I.Q. had its highest loading -- .47 on Factor III, Group Recognition - A.

5. The environment of the home does not significantly affect the child's concept of number. No significant difference was found to exist in the number of concepts of children from high socio-economic homes and those in low socio-economic homes. This finding supports the theory proposed earlier that when the child attends school the effects of the home environment will continue to decrease because all children in school are exposed to similar activities without reference to their socio-economic status.

This conclusion is very important since Brace (1963) found that socio-economic status had a highly significant affect on the pre-school children's concept of number.

6. All three of Piaget's stages in the development of number were observed in the children tested. The three stages were exemplified by (a) those students who showed an almost complete lack of knowledge of a particular concept; (b) those who had "intuitive" ideas of the concept being measured; and (c) those who demonstrated a "concrete operational" knowledge of a particular concept. It was also observed but not studied that a particular child may be functioning at different stages in different concepts.

IMPLICATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

The results of this study have educational and psychological implications. Educationists responsible for the construction of the arithmetic curriculum should be cognizant of the importance of chronological age in determining when formal instruction in the basic concepts of number should be introduced. In view of the results of this study, it would appear that the arithmetic program, for the child who has not reached seven years of age, should consist of familiarizing him with concrete objects and activities which will prepare him for formal instruction. These activities would increase his ability to discriminate and synthesize; they are basic requirements for continuous intellectual development. After this program, the formal introduction of the basic processes of cardinationand ordination would be consistent with his

intellectual development of an understanding of number. The basic operations of addition, subtraction, multiplication and division are introduced only after the child has developed an adequate knowledge of the basic concepts of number. Through this process, his understanding of number will be developed in a sequential and systematic manner. This approach should lessen the number of frustrations which can develop into a dislike for mathematics.

The results of this study showed that some children after nine months of arithmetic instruction still exhibit a complete lack of understanding of some of the concepts of number. One may therefore assume that for some students at least one year and possibly more should be used to develop an understanding of number.

This study has presented tentative answers for some important questions, however, it has also opened up new areas which require further research. Some suggestions for further research are:

1. The hypothesis that the number concepts of girls are superior to boys due to greater reinforcement should be tested. Future studies should be conducted with students who are taught for example by female teachers and programmed instruction.
2. Additional studies are required to determine what effect the variables I.Q. and socio-economic status have on a child's ability to develop an understanding of number. One such study could involve the categorization of students according to three levels of I.Q. and socio-economic status. This design would permit a more accurate determination of the influence of intelligence and socio-

economic status.

3. A study is required in which an analysis of variance is conducted with the scores of each individual concept.

4. Can a child develop an understanding of the number concepts at an earlier age than this study suggests? Further research is required in this area.

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APPENDIX A

APPENDIX A

A TEST OF THE BASIC CONCEPTS OF NUMBER

Subtest I - Cardinal Number

(Materials; 8 cards of coloured buttons containing groups of 3 to 10, and 75 small blocks.)

1. Point to a card which has three buttons on it.
2. Point to a card which has seven buttons on it.
3. Point to a card which has ten buttons on it.
4. Point to a card which has nine buttons on it.

Present the child with groups of blocks ranging in size from two to nine and arranged in random patterns, both within and among groups.

5. Point to a group which has only five blocks in it.
6. Point to a group which has only eight blocks in it.
7. Point to a group which has only ten blocks in it.
8. Point to a group which has only twelve blocks in it.

Subtest II - Ordinal Number

(Materials: 12 toy horses, a toy ladder and a toy fireman, a simulated parking lot of cardboard with fifty toy cars.)

Using the fireman and ladder:

9. Place the fireman on the fourth step on the ladder.
10. Place the fireman on the ninth step on the ladder.

Arrange the 50 toy cars on the simulated parking lot in five rows of ten cars each. Suppose you are entering the lot from the top left

corner (indicate starting position).

11. In the fourth row, touch the second car.

12. In the third row, touch the eighth car.

Suppose you were entering the parking lot from this bottom right corner (indicate starting position)

13. In the second row, touch the sixth car.

14. In the fifth row, touch the ninth car.

Arrange the twelve toy horses in single file as if running a race.

15. Touch the horse that is in second place.

16. Touch the horse that is in ninth place.

17. Touch the horse that is in fifth place.

18. Touch the horse that is in eleventh place.

Subtest III - Conservation of Number

19. (Materials: nine toy horses, 9 small toy riders)

Arrange the nine horses and nine riders in a one-to-one correspondence; after the child is convinced that each horse has a rider, move all the horses into a large pile and all the riders into a small pile.

Are there more horses now than riders?

20. (Materials; 12 red blocks and 12 blue blocks)

Present the child with a group of twelve red blocks and a group of twelve blue blocks, or equal size, arranged close together and in one-to-one correspondence. Again make sure that the child sees that there are as many red blocks as blue blocks. Then spread the blue blocks

apart, leaving a space approximately equal to the width of one block between them.

Are there as many red blocks now as there are blue blocks?

21. (Materials; 7 toy soldiers, 7 toy cars, 7 toy dogs)

Ask the child to give each soldier a toy car; after the child is convinced that each soldier received a car, move all the cars on a pile. Then, ask the child to give each soldier a toy dog; again after the child is convinced that each soldier received a toy dog, move all the dogs on a separate pile.

Are there more cars than dogs?

22. (Materials; 2 transparent cylindrical glasses of the same size and shape, 12 small beads, 12 large marbles approximately twice the size of the beads).

Ask the child to place a bead in his glass at the same time as I (experimenter) place a marble in my glass. This procedure is repeated twelve times.

Do I have more marbles than you have beads?

23. (Materials; 2 transparent cylindrical glasses of the same size and shape, 30 marbles of the same size, 1 large vessel).

Ask the child to place a marble in his glass at the same time as I place a marble in my glass. Repeat the procedure fifteen times.

Do you have the same number of marbles as I do?

24. (Materials; same as Question 23).

After the child is convinced that he has the same number of marbles as I do (15 marbles each), pour my fifteen marbles into a large vessel.

Now, do you have more marbles than I do?

25. (Materials; 2 large cylindrical glasses of equal size and shape, 1 large cylindrical glass taller, but smaller in diameter than the other two large ones, red and blue coloured liquid.)

Present the child with two uniform glasses each containing an equal quantity of coloured liquid, one with red and the other with blue liquid. When the child is convinced that the quantity of liquid in each glass is the same, pour the blue liquid into the glass that is taller and smaller in diameter.

Is there more blue liquid now than red liquid?

26. (Materials; 2 medium cylindrical glasses of equal size and shape, 1 short glass but larger in diameter than the other two glasses, 4 small cylindrical glasses of equal siz and shape and red and blue coloured liquid.)

Present the child with two medium sized glasses each containing an equal quantity of coloured liquid, one with red and the other with blue liquid. When the child is convinced that the quantity of liquid in each glass is the same, pour all the red liquid into the four small glasses and pour all the blue liquid into the one large glass.

Is there more red liquid now than blue liquid?

Subtest IV - Arithmetical Operations

27. (Materials; 14 red poker chips).

Seven chips are arranged in one group while the other seven chips are arranged in a group which is subdivided into a group of four and

three chips.

Suppose these two groups of chips (4 + 3 = 7 chips) are yours and this group (7 chips) is mine; without counting, do you have the same number of chips as I do?

28. (Materials; 16 one-inch cubical blocks).

Eight blocks are arranged into a large group with three subdivisions, two groups of three and one group of two blocks; the other eight blocks are arranged into one large group with two subdivisions of four blocks each.

Suppose your father gave you these blocks (3 + 3 + 2 = 8) and my father gave me these blocks (4 + 4 = 8), without counting, did you receive more blocks than I did?

29. (Materials; 8 red chips, one inch in diameter, 9 green chips one inch in diameter).

The eight red chips are arranged into one large group while the nine green chips are arranged into one large group of nine chips with two subgroups, a group of five and a group of four chips.

Suppose I found these red chips (8 chips) and you found these green chips (5 + 4 = 9 chips), without counting, did I find as many chips as you did?

30. (Materials; 18 jacks).

One group of nine jacks had two subgroups, a group of eight jacks plus one jack; the other group of nine jacks was arranged into three equal groups.

Suppose I bought these jacks (8 + 1 = 9) and you bought these

jacks (3 + 3 + 3 = 9), without counting, do I have more jacks than you?

31. (Materials; 15 blue poker chips, 15 red poker chips).

One group, the 15 blue chips, are divided into three equal groups with the same domino pattern; the other group, 15 red chips, are subdivided into two groups, one of nine and the other of six chips.

Suppose these blue chips (5 + 5 + 5 = 15) or (3 x 5 = 15) are yours and these red chips (9 + 6 = 15) are mine; without counting, do I have more chips than you do?

32. (Materials; 2 toy soldiers, 22 large wooden sticks, four inches in length).

The 22 sticks are arranged into four large groups; one group has four sticks, a second group has five sticks divided into two subgroups, a group of two sticks and one with three sticks, a third group has six sticks, divided into two subgroups, a group of two sticks and one with four sticks; the last group has seven sticks, divided into two subgroups, a group of four sticks and a group of three sticks.

Without counting, point to a group which has just enough sticks so that I can give each soldier three sticks (2 x 3 = 6).

33. (Materials; 5 toy soldiers, 51 red chips, one inch in diameter).

The 51 chips are arranged in five large groups; one group has nine chips divided into three equal subgroups, a second group has seven chips divided into two subgroups, a group of three chips and one of four chips, a third group has ten chips divided into two equal domino arranged subgroups, a fourth group has eleven chips divided into two

subgroups, a group of five chips and a group of six chips, the fifth group has fourteen chips divided into two subgroups, a group of five chips and one of nine chips.

Without counting, point to one large group which has just enough chips so that I can give each soldier (5 soldiers) two chips ($2 \times 5 = 10$).

34. (Materials; 7 white poker chips).

The seven chips are divided into two groups, a group of three chips and one of four chips.

Without counting, can you tell me how many white chips there are here, and the number story?

35. The child is told that he must listen carefully to the question because no materials will be used in this item.

Suppose you had six cookies and you gave me four, can you tell me how many cookies you had left, and the number story?

36. In this item the child is again warned to listen carefully because no concrete materials will be used to explain the question.

Suppose one boy had six apples and the other boy had three apples; can you tell me how many apples the two boys had altogether, and the number story?

Subtest IV - Place Value in the Decimal System of Notation

(Materials; several bundles of 10 sticks and 12 single sticks).

37. Using bundles of 10 sticks and single sticks, make a group of 14 sticks.

38. Using bundles of 10 sticks and single sticks, make a group of 19 sticks.

39. Using bundles of 10 sticks and single sticks, make a group of 45 sticks.

40. Using bundles of 10 sticks and single sticks, make a group of 97 sticks.

41. Using bundles of 10 sticks and single sticks, make a group of 113 sticks.

APPENDIX B

TABLE XVI

INTERCORRELATIONS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	T.S.	Sex	Age	Pgm.	S.E.S.	I.Q.
1	1.000	183	282	158	338	142	089	094	363	131	195	189	118	076	316	157	225	118	202	134	217	121	189	172	104	121	172	169	131	138	087	115	094	234	145	189	195	153	209	172	035	115	110	110	113	113	113
2		1.000	161	567	168	315	318	402	206	207	291	142	311	294	-042	212	112	087	183	110	022	103	078	-037	122	159	269	189	093	126	361	071	289	135	-015	014	226	194	070	147	-007	295	183	097	224	000	056
3			1.000	483	230	279	020	040	262	318	105	009	-016	-122	-028	252	163	202	033	255	-030	212	174	214	152	212	214	201	097	193	159	264	187	180	291	339	105	164	220	293	124	337	099	026	-182	036	036
4				1.000	203	463	296	222	247	135	137	-005	115	196	-003	385	106	221	161	100	-048	186	054	110	079	186	225	146	082	173	225	150	169	199	157	115	260	197	059	225	221	362	062	062	132	-026	-132
5					1.000	163	180	195	339	-034	096	180	184	045	145	-062	-063	-067	-088	229	031	109	085	327	226	276	236	315	136	153	172	175	110	160	344	180	096	-070	020	055	103	258	158	158	-083	000	167
6						1.000	264	145	117	199	133	109	220	093	123	409	180	220	-082	063	-162	086	225	207	017	137	151	239	147	031	192	148	041	144	073	167	252	169	124	151	065	405	160	108	180	-026	-026
7							1.000	376	065	061	104	008	191	073	-123	296	077	140	203	093	100	120	241	073	189	223	241	149	165	021	391	006	271	053	139	125	163	153	121	185	117	262	-006	251	231	128	282
8								1.000	-010	104	191	038	238	202	-023	330	229	136	110	032	249	064	211	105	139	319	216	127	464	-042	557	106	330	-053	177	038	191	136	269	161	100	258	046	148	127	026	229
9									1.000	181	128	167	060	113	030	-034	066	060	035	100	-053	156	217	183	110	068	280	268	002	199	057	-037	080	305	308	117	332	051	045	183	096	229	213	125	-044	221	-044
10										1.000	146	062	094	034	-066	296	012	144	-005	-104	-138	-038	119	-177	-014	-037	099	130	130	-134	144	-029	-050	235	118	177	088	323	021	155	-092	072	-123	-224	000	051	101
11											1.000	312	145	140	-023	260	289	087	085	161	053	-015	-016	000	072	043	063	171	146	059	030	130	015	093	209	115	267	303	173	189	077	130	-188	217	-116	058	058
12												1.000	001	157	417	-055	128	-056	-066	077	107	-040	032	-019	098	130	043	149	119	093	-007	-014	-077	-063	007	096	049	039	087	043	-117	043	-271	127	-085	199	-028
13													1.000	425	057	221	-038	098	-060	016	118	-129	172	033	079	072	197	061	-109	-010	168	124	034	-008	-116	058	-029	089	087	-022	-060	224	-024	-024	401	100	050
14														1.000	069	082	000	-115	-004	-092	087	-131	096	-088	-085	030	029	047	039	-110	035	-044	-017	-091	-035	-088	202	325	234	206	-018	118	-010	-064	459	189	027
15															1.000	164	317	054	080	184	299	066	147	199	172	225	112	187	014	039	112	048	-104	032	-030	057	059	153	092	112	-030	207	030	-049	-040	119	-119
16																1.000	370	592	161	-007	017	027	115	052	026	080	167	203	189	010	225	097	222	063	102	115	260	418	312	283	128	309	062	114	132	036	-079
17																	1.000	401	015	089	122	100	057	013	087	038	-123	135	139	-026	323	138	038	000	129	270	289	223	329	355	044	263	-013	238	125	125	-188
18																		1.000	057	067	-005	12[illegible]	115	087	180	172	087	223	144	041	220	174	187	056	142	115	260	089	267	197	117	324	126	126	050	050	-150
19																			1.000	368	500	307	201	276	223	307	212	067	114	034	129	104	110	108	053	134	221	145	262	276	-035	220	074	015	029	029	029
20																				1.000	310	598	250	615	574	547	177	202	101	249	230	199	186	183	243	193	279	184	278	338	030	504	157	157	-229	076	127
21																					1.000	315	385	327	298	437	060	045	048	013	213	227	001	-015	-024	107	124	131	232	260	-055	227	079	-044	153	031	092
22																						1.000	358	541	609	497	159	186	014	164	253	098	115	068	265	187	217	213	161	323	199	500	102	001	-125	025	125
23																							1.000	415	269	415	291	272	177	209	345	043	038	-068	007	225	180	098	291	353	185	441	184	070	085	085	256
24																								1.000	443	542	345	265	210	356	225	116	105	175	282	167	252	092	235	256	203	508	475	038	-164	-055	109
25																									1.000	710	279	139	138	092	369	152	241	124	252	212	246	208	320	279	245	554	149	048	-025	025	075
26																										1.000	323	240	217	215	356	299	165	068	265	244	333	265	401	323	110	550	203	152	-025	-025	176
27																											1.000	265	044	523	282	016	216	245	169	043	252	092	039	107	106	398	038	147	-055	-109	055
28																												1.000	239	165	077	152	017	229	035	149	171	183	153	088	017	368	172	172	-135	027	189
29																													1.000	072	300	021	258	105	170	119	146	058	324	210	087	274	181	080	-051	101	253
30																														1.000	105	123	062	131	053	-023	177	-011	-012	078	072	225	184	082	-051	000	-153
31																															1.000	084	400	174	280	169	268	081	294	394	126	497	071	329	207	052	103
32																																1.000	106	172	019	157	072	018	133	071	037	250	-049	201	075	-025	-025
33																																	1.000	012	282	-020	191	083	208	105	100	360	046	250	127	-076	025
34																																		1.000	157	150	462	252	125	245	149	300	276	146	032	-032	096
35																																			1.000	242	447	244	263	338	148	329	187	239	-310	052	155
36																																				1.000	246	277	427	477	185	384	070	127	-142	028	199
37																																					1.000	605	588	630	179	477	274	274	115	-058	000
38																																						1.000	483	549	227	333	139	-018	105	-052	052
39																																							1.000	627	167	493	166	226	120	000	000
40																																								1.000	203	562	202	202	109	109	055
41																																									1.000	302	318	141	-133	044	044
42																																										1.000	301	251	075	075	025
43																																											1.000	-001	025	025	-025
44																																												1.000	025	025	025
45																																													1.000	000	-100
46																																														1.000	150
47																																															1.000

42 Total Score - Two groups: (a) below 27.5; (b) above 27.5
43 Sex
44 Age - Two groups: (a) below 7 years; (b) 7 years and above
45 Program - Two groups: (a) Ginn Series ; (b) Gage Series
46 Socio-economic status: (a) High S.E.S.; (b) Low S.E.S.
47 I.Q. - Two groups - (a) below 108.5; (b) above 108.5

TABLE XVII

APPENDIX B

ELEVEN UNROTATED FACTORS OBTAINED FROM PRINCIPAL AXIS FACTOR ANALYSIS OF FORTY-ONE TEST ITEMS AND REFERENCE VARIABLES

	1	2	3	4	5	6	7	8	9	10	11
1	-385	082	-114	-082	-433	026	015	085	-066	134	379
2	-399	472	083	-407	-028	-027	-012	-092	371	007	-082
3	-446	020	-467	-057	-019	-033	-173	-343	014	-174	108
4	-453	349	-250	-268	095	-267	-105	-145	175	-113	-105
5	-363	-132	-201	-472	-214	220	072	196	-118	-284	100
6	-400	359	-208	-168	-022	-204	-132	-126	-267	-183	-109
7	-404	267	218	-280	257	190	079	-055	-014	021	023
8	-440	323	354	-202	238	333	-014	009	-102	149	-013
9	-346	007	-423	-243	-276	-047	258	196	003	073	012
10	-164	382	-264	034	-113	077	200	-596	008	144	045
11	-293	296	-122	089	-283	318	-070	046	329	033	-069
12	-127	095	042	-041	-644	320	-195	139	083	043	-286
13	-188	383	314	-333	-172	-229	099	-013	-118	-320	032
14	-140	457	337	-042	-274	-168	425	196	016	-134	-223
15	-228	-094	145	125	-580	-107	-292	117	-218	122	-131
16	-438	515	-062	180	124	-128	-292	-200	-162	216	-058
17	-355	291	-004	443	-121	099	-431	174	-130	012	019
18	-374	235	-129	204	203	-121	-450	-079	-246	125	012
19	-370	-185	301	101	-013	-130	014	-172	325	376	286
20	-571	-501	078	009	-084	019	-099	-056	221	-105	-058
21	-347	-266	598	120	-235	-083	-038	-059	069	108	278
22	-528	-478	087	028	058	-078	-052	-255	151	-097	-190

TABLE XVII

APPENDIX B (Cont'd.)

ELEVEN UNROTATED FACTORS OBTAINED FROM PRINCIPAL AXIS FACTOR ANALYSIS OF FORTY-ONE TEST ITEMS AND REFERENCE VARIABLES

	1	2	3	4	5	6	7	8	9	10	11
23	-505	-160	273	-115	-083	-081	120	-193	-346	090	-036
24	-580	-555	013	-126	-006	-169	007	029	-176	050	-060
25	-588	-398	205	-012	052	016	-074	-086	127	-189	-207
26	-674	-358	280	-069	-011	008	-066	-083	056	-107	-068
27	-450	-075	-101	-485	007	-160	017	022	152	329	-084
28	-419	-007	-189	-198	-247	065	-056	-081	-255	160	077
29	-368	084	065	-004	139	407	050	007	-318	334	016
30	-278	-243	-202	-333	-068	-270	-146	212	105	316	-073
31	-587	145	249	-135	303	178	-108	089	-052	028	-002
32	-276	025	018	-045	-046	-057	-366	-011	090	-379	499
33	-361	110	070	-184	468	158	-135	181	169	062	-004
34	-352	001	-406	-008	-017	-150	275	100	116	096	407
35	-459	-125	-347	100	159	362	156	137	127	-061	-117
36	-442	-043	-161	303	-101	195	080	-216	-094	-322	184
37	-646	125	-165	322	039	-117	271	299	222	123	-003
38	-492	249	-043	445	-090	-147	329	-123	205	022	-157
39	-605	095	145	489	035	040	102	137	-020	-044	047
40	-666	062	010	459	-001	-031	225	036	069	-039	-014
41	-300	-103	-201	089	270	-042	137	109	-184	-174	-237
Total Score	-832	-064	009	-012	093	-133	-050	025	-080	-156	-127
Sex	-304	-269	-181	067	202	-329	280	244	-477	093	064
Age	-310	083	-082	-025	155	308	-206	535	087	-106	179
Program	-060	480	456	-069	051	-417	130	216	-045	-133	205
S.E.S	-080	075	067	006	-291	220	094	043	-254	-022	-132
I.Q.	-151	-103	164	-181	024	514	433	-280	-166	-024	186

TABLE XVIII

APPENDIX B

ELEVEN ROTATED FACTORS OBTAINED FROM PRINCIPAL AXIS FACTOR ANALYSIS OF FORTY-ONE TEST ITEMS AND REFERENCE VARIABLES

	1	2	3	4	5	6	7	8	9	10	11
1	-029	174	055	408	050	066	342	017	082	254	368
2	097	054	326	313	127	417	-117	514	137	007	-009
3	253	098	-027	270	309	-125	-077	104	363	-214	392
4	175	099	096	374	369	299	-228	251	191	-226	086
5	217	-081	188	410	231	090	271	030	-059	-302	404
6	094	034	101	212	448	319	090	-021	222	-317	152
7	125	048	607	080	040	230	-074	081	094	-022	034
8	092	062	762	-014	122	176	085	090	017	068	-031
9	015	220	030	630	-101	033	194	010	068	-182	097
10	-147	168	084	100	150	006	003	175	744	-000	022
11	-002	225	167	191	041	-015	215	561	037	-026	099
12	060	012	-047	077	010	-012	662	477	-093	002	-040
13	048	-080	119	034	024	728	108	046	084	-045	143
14	-096	313	056	000	-146	691	235	109	005	-036	-224
15	210	008	-209	075	269	065	621	003	-128	217	-008
16	-031	222	269	046	727	110	019	052	224	039	-028
17	-024	335	100	-151	571	-063	304	106	-194	054	198
18	064	103	179	013	717	-078	-008	102	022	-016	104
19	316	206	116	139	018	-046	-151	047	076	653	-018
20	738	169	008	139	-048	-140	049	081	-074	105	153
21	420	103	081	-105	-067	162	176	-077	-070	652	131
22	782	125	002	048	024	-112	-066	-002	099	035	002
23	449	040	256	105	065	188	264	-312	226	145	-022
24	680	060	117	310	035	-100	023	-379	-073	010	026

TABLE XVIII (Cont'd)

APPENDIX B

ELEVEN ROTATED FACTORS OBTAINED FROM PRINCIPAL AXIS FACTOR ANALYSIS OF FORTY-ONE TEST ITEMS AND REFERENCE VARIABLES

	1	2	3	4	5	6	7	8	9	10	11
25	778	149	133	002	-000	-002	023	043	-059	-015	048
26	757	139	224	077	017	065	095	-015	-032	124	120
27	301	-063	222	649	065	070	-062	087	036	080	-146
28	142	-002	163	389	170	-043	337	-075	206	006	170
29	010	132	604	070	102	-199	260	-156	101	043	-047
30	250	-104	006	600	144	-089	-106	-048	-216	051	-137
31	296	143	630	056	207	131	-022	019	-114	006	077
32	191	-043	-011	001	177	111	-105	083	-116	131	695
33	153	073	531	125	137	-016	-288	122	-237	-068	015
34	-064	384	-018	521	-051	-046	-165	-128	109	067	306
35	222	431	258	193	-090	-332	005	097	-027	-328	103
36	240	396	042	-114	043	-097	142	-040	275	-117	454
37	144	792	116	326	119	046	-044	010	-112	024	-013
38	155	739	-035	023	140	160	005	113	265	035	-128
39	235	678	211	-115	193	062	123	-090	-073	103	120
40	301	750	123	014	144	054	066	-053	071	046	075
41	235	251	092	054	075	-036	-063	-246	-041	-409	-046
Total Score	601	338	249	223	289	175	052	-091	-011	-141	143
Sex	139	234	015	304	056	007	002	-729	-078	-127	-023
Age	-031	200	360	138	065	-102	027	127	-504	-128	332
Program	-188	106	113	-064	103	755	-129	-113	-160	209	035
S.E.S.	-023	045	093	-044	-064	061	454	-012	070	-092	-017
I.Q.	083	008	472	-078	-477	-072	153	-104	392	022	153

APPENDIX C

APPENDIX C

TABLE XIX
DATA FOR THE TOTAL SAMPLE

AGE Yrs. Mos.	SEX	S.E.S.*	PGM.+	I.Q.	FACTOR I	II	III	IV	V	Total (20 Items)	Total (41 Items)
6 - 5	F	H	S	102	2	3	1	3	1	10	25
	F	L	G	119	0	4	1	1	1	7	21
	F	H	G	132	0	0	1	3	0	4	15
	M	H	S	94	0	3	0	1	3	4	18
6 - 6	F	L	S	120	0	3	3	2	1	9	23
	M	L	S	123	5	4	4	3	1	17	31
	M	L	G	129	5	4	2	2	2	15	28
	M	H	G	102	3	0	0	1	0	4	13
	M	L	G	108	4	4	3	3	3	17	36
6 - 7	F	L	S	108	5	4	1	3	3	16	31
	F	H	S	106	1	0	0	2	0	3	16
	M	L	S	111	1	0	0	1	0	2	10
	M	L	S	103	0	4	3	1	3	11	25
	M	H	S	122	0	4	3	1	3	11	26
	M	H	G	116	3	3	3	3	3	15	32
	M	L	G	111	2	3	4	3	2	14	27
6 - 8	F	L	S	99	0	1	2	2	3	8	14
	F	L	S	111	2	0	3	2	2	9	24
	F	L	G	111	2	0	1	3	0	6	18
	F	H	G	117	0	0	0	1	1	2	16
	F	L	G	101	5	4	1	3	1	14	28
	M	H	S	108	5	4	1	3	1	14	30
	M	H	G	104	4	4	0	2	3	13	24
6 - 9	F	H	S	113	5	4	1	2	3	15	34
	F	H	S	116	5	4	5	2	3	19	38
	F	L	G	107	3	4	0	3	3	13	28
	F	H	G	109	5	3	1	0	2	11	21
	F	H	G	105	5	3	5	3	3	19	38
	M	H	S	113	5	4	3	3	1	16	31
	M	L	S	104	3	0	0	1	1	5	15
6 - 10	F	H	G	87	0	0	3	0	3	6	17
	M	L	G	92	5	4	3	3	1	16	32
	M	H	G	114	5	4	3	3	0	15	31
6 - 11	F	H	S	111	1	4	1	0	3	9	27
	F	L	S	104	0	3	4	0	2	9	25
	F	L	G	107	5	4	0	1	3	13	33

APPENDIX C

TABLE XIX (Cont'd.)

AGE Yrs. Mos.	SEX	S.E.S*	PGM.$^+$	I.Q.	FACTORS					Total (20 Items)	Total (41 Items)
					I	II	III	IV	V		
6 - 11	M	H	S	111	2	1	1	3	1	8	25
	M	L	S	107	4	4	1	2	3	14	28
	M	L	G	102	1	4	1	3	2	11	25
	M	H	G	111	5	0	0	2	2	9	22
7 - 0	F	H	S	107	5	3	4	2	3	17	34
	F	H	S	98	0	4	1	1	1	7	17
	F	L	S	108	5	4	3	3	3	18	37
	F	L	G	109	4	3	2	1	3	13	27
	F	H	G	86	0	1	1	0	1	3	11
	M	L	S	108	3	4	1	3	2	13	32
	M	H	S	102	4	4	5	3	3	19	38
	M	L	G	108	2	4	2	1	2	11	22
	M	H	G	94	4	3	1	2	2	12	28
7 - 1	F	L	S	93	1	3	2	3	3	12	23
	F	L	G	102	0	2	0	3	2	7	21
	F	H	G	114	5	4	5	3	2	19	37
	M	H	S	125	2	3	5	3	3	16	28
	M	L	S	112	5	3	3	2	1	14	31
	M	L	G	106	5	4	3	3	3	18	34
	M	H	G	127	5	4	2	3	3	17	32
	M	H	G	112	5	4	3	3	2	17	33
7 - 2	F	H	S	112	1	4	3	2	2	12	30
	F	L	S	123	3	4	5	3	3	18	30
	F	H	G	112	1	4	1	3	3	12	27
	F	L	G	123	4	4	0	3	0	11	24
	F	H	G	114	5	2	3	2	1	13	24
	M	H	S	106	1	4	1	3	3	12	28
	M	L	S	102	2	4	5	3	3	17	33
	M	H	G	97	2	3	1	3	3	12	29
	M	L	G	114	5	4	0	2	0	11	29
7 - 3	F	H	S	112	0	4	1	0	1	6	19
	F	L	S	125	5	4	5	2	3	19	38
	F	L	G	91	0	0	0	3	3	6	20
	M	L	S	106	3	2	1	3	2	11	19
	M	H	S	93	5	4	5	3	3	20	38
7 - 4	F	L	S	104	0	1	2	3	0	6	19
	F	H	S	104	4	2	3	1	3	13	30

APPENDIX C

TABLE XIX (Cont'd.)

AGE		SEX	S.E.S.*	PGM.+	I.Q.	FACTORS					Total	Total
Yrs.	Mos.					I	II	III	IV	V	(20 Items)	(41 Items)
7 -	4	F	L	G	118	5	1	4	2	0	12	26
		M	H	S	116	3	4	5	3	2	17	34
		M	L	S	119	2	4	2	2	2	10	29
		M	L	G	118	4	2	5	2	3	16	33
		M	H	G	121	5	4	5	2	3	19	37
7 -	5	F	H	G	121	1	2	4	2	3	12	29
		M	L	G	102	0	2	2	2	3	9	27

*S,E,S. (Socio-economic Status) - H represents a child from a high socio-economic group; L represents a child from a low socio-economic group.

+Program - S represents a child from the "Seeing Through Arithmetic" Series by Gage- instructional group; G represents a child from the "Arithmetic We Need" Series by Ginn instructional group.

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